Bending Moment :-

$BM_B = 0$ KNm

$BM_C = 7 \times 3 = 21$ KNm

$BM_E = 7 \times 7 - 5 \times 4 - 4 \times 2 = 21$ KNm

$BM_A = 7 \times 10 - 5 \times 7 - 4 \times 5 - 5 \times 3 = 0$ KNm

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Mechanical properties of materials [2 Marks]

(1) Hardness - The property of material by which it resist the local deformation ~~when~~ when undergoes an impact.

- It is the state of material, being for which it can withstand the load.

(2) Creep - These is the permanent change in shape and size of a material under the application of load or temperature.

- It is the temperature dependent (time).

- Creep at different temperature for different materials

(3) Rigidity - These is defined as the property possessed by a solid body to change its shape

- These is the property of material to resist the bending.

(4) Fatigue :- These is the property of a material subjected to repeated cyclic load to resist the deterioration.

(5) Durability :- The ability of a material to remain serviceable during the useful time without damaging the material.

- It represents how long the material works.

(6.) Elasticity & plasticity

When the force is applied on a material it undergoes deformation but after ~~start~~ removing the force it return back to its original shape. This property of material is called as Elasticity

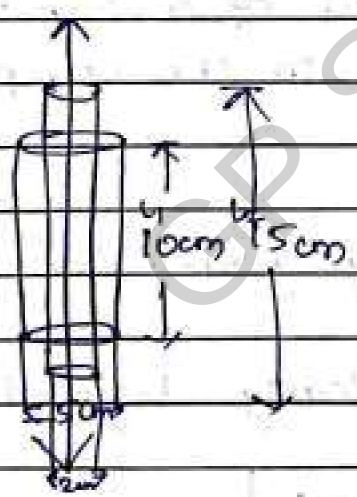
When the force is ~~f~~ applied on a material

Poisson's Ratio :-

- Poisson's Ratio is defined as "the ratio between the lateral strain to the longitudinal strain"
- It is denoted by symbol (μ).

$$\boxed{\mu = \frac{\text{lateral strain}}{\text{longitudinal strain}}}$$

$$\mu = \left| \frac{\text{lateral strain}}{\text{longitudinal strain}} \right|$$



$$\mu = \left| \frac{-3}{5} \right|$$

$$= \frac{3}{5}$$

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MODULUS OF RIGIDITY :-

1.— It is also known as Shear modulus

2.— It is the ~~major~~ measure of the rigidity of body, given by the ratio of shear stress to shear strain

3.— It is denoted by the symbol 'G'

$$\text{Shear modulus} = \frac{\text{Shear stress}}{\text{Shear strain}} \quad \frac{\tau}{e}$$

Bulk modulus

- The Bulk modulus of a Substance is a major of how resistant to the compression of the substance

- It is denoted by symbol 'K'

- It is mathematically defined as

$$\boxed{K = -V\left(\frac{dP}{dV}\right)}$$

Teacher's Signature..........

Relationship between :-

$$\boxed{E = 2G(1+\mu)}$$

$$\boxed{E = 3K(1-2\mu)}$$

Q. A cylindrical bar is 20mm diameter and 750mm long. During test, longitudinal strain was found 3 times of the lateral strain calculate the modulus of rigidity and Bulk modulus. If the $E = 2 \times 10^5 \, N/mm^2$.

Lateral strain = x

longitudinal strain = $3x$

$$\mu = \frac{x}{3x}$$

$$\mu = \frac{1}{3}$$

$$\mu = 0.33$$

$$2 \times 10^5 = 2G(1 + 0.33)$$

$$2G = \frac{2 \times 10^5}{1.33}$$

$$G = \frac{2 \times 10^5}{1.33 \times 2}$$

$$G = 75.18 \times 10^3 \, N/mm^2$$

$$2 \times 10^5 = 3K(1 - 2 \times \tfrac{1}{3}$$

$$3K = \frac{2 \times 10^5}{\cancel{0.34} \times 3}$$

$$K = \cancel{196.07 \times 10^2}$$

$$2 \times 10^5 \, N/mm^2$$

Teacher's Signature.......................

(ii) Also find the change volume when volumetric stress of $100 N/mm^2$ is applied.

$$K = \frac{\text{volumetric stress}}{\text{volumetric strain}}$$

$$2\times10^5 N/mm^2 = \frac{100 N/mm^2}{\frac{dv}{v}}$$

$$dv = \frac{100}{2\times10^5} \times v$$

$$dv = \frac{100}{2\times10^5} \times l \times A \quad \left[A = \frac{\pi}{4} \times 20^2\right]$$

$$= \frac{\pi}{4} \times 20^2 = 314.159$$

$$dv = \frac{100}{2\times10^5} \times 750 \times 314.159$$

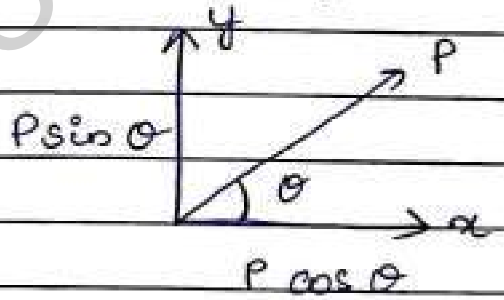
$$dv = 117.80 mm^3$$

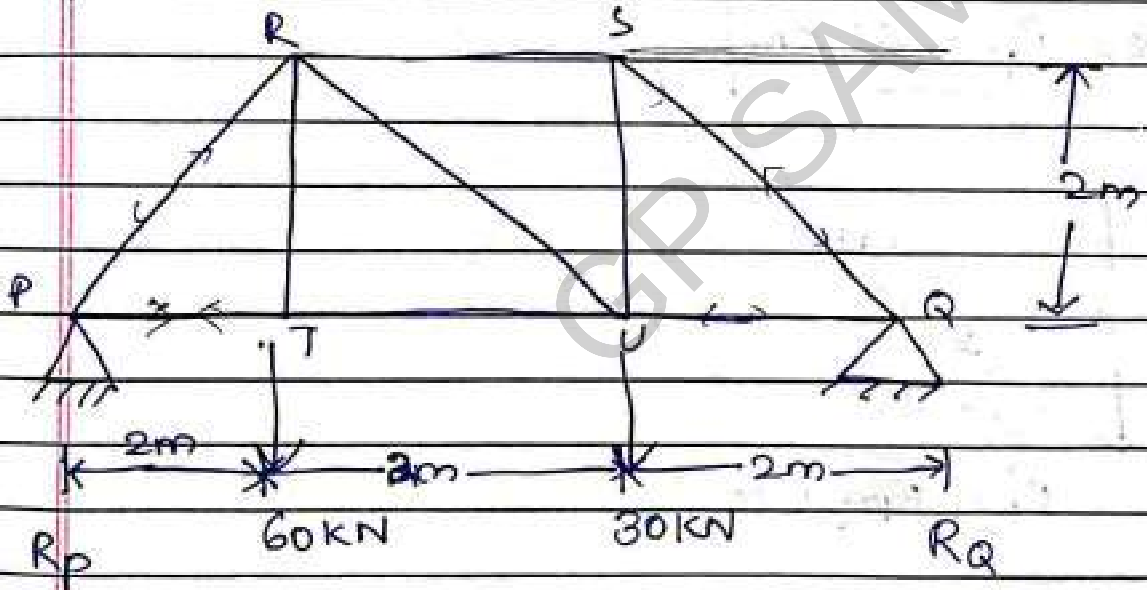
Analysis of trusses :-

Define truss :-

A framework, typically consisting of posts struts & supporting a roof or bridges.

Resolution of forces :-





$$\Sigma f_y = 0$$

$$\rightarrow R_p - RT + RQ - SU = 0$$

$$R_p + R_Q - 60 - 30 = 0$$

$$R_p + R_Q = 90$$

Take $M_P = 0$

$(-R_Q \times 6) + (SU \times 4) + (RT \times 2)$

$(-RQ \times 6) + (30 \times 4) + (60 \times 2)$

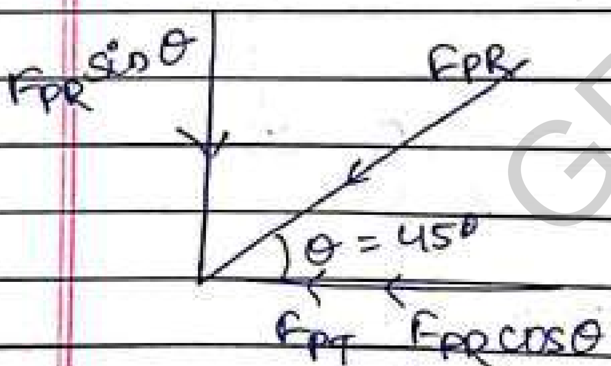
$-6RQ + 120 + 120$

$RQ = \dfrac{240}{6}$

$\boxed{RQ = 40 kN}$

$R_P + RQ = 90 kN$

$\boxed{R_P = 50 kN}$



$\Sigma f_y = 0$

$\Rightarrow +R_P - F_{PR} \sin\theta = 0$

$\Rightarrow R_P = F_{PR} \sin\theta$

$\Rightarrow 50 = F_{PR} \sin 45^\circ$

Teacher's Signature..................

$$\Rightarrow \frac{50}{\sin 45} = F_{PR}.$$

$$\Rightarrow \boxed{70.71\,KN = F_{PR}}. \quad [\text{Tension}]$$

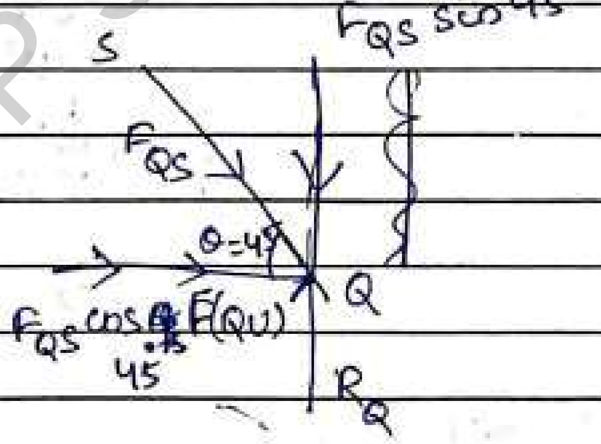
$$\Sigma f_x = 0.$$

$$F_{PT} + \cancel{R_B}\, F_{PR}\cos\theta =$$

$$F_{PT} + 70.71 \times \frac{1}{\sqrt{2}}$$

$$\boxed{F_{PT} = (-)49.99\ (C)}$$

FBD of point Q



$$\Sigma f_y = 0$$

$$R_Q - F_{QS}\sin 45 = 0$$

$$R_Q = F_{QS}\sin 45°$$

$$F_{QS} = \frac{40}{\sin 45}.$$

$$F_{QS} = 40\sqrt{2}.$$

$$\boxed{F_{QS} = 56.56\ kN}\ (\text{Tension})$$

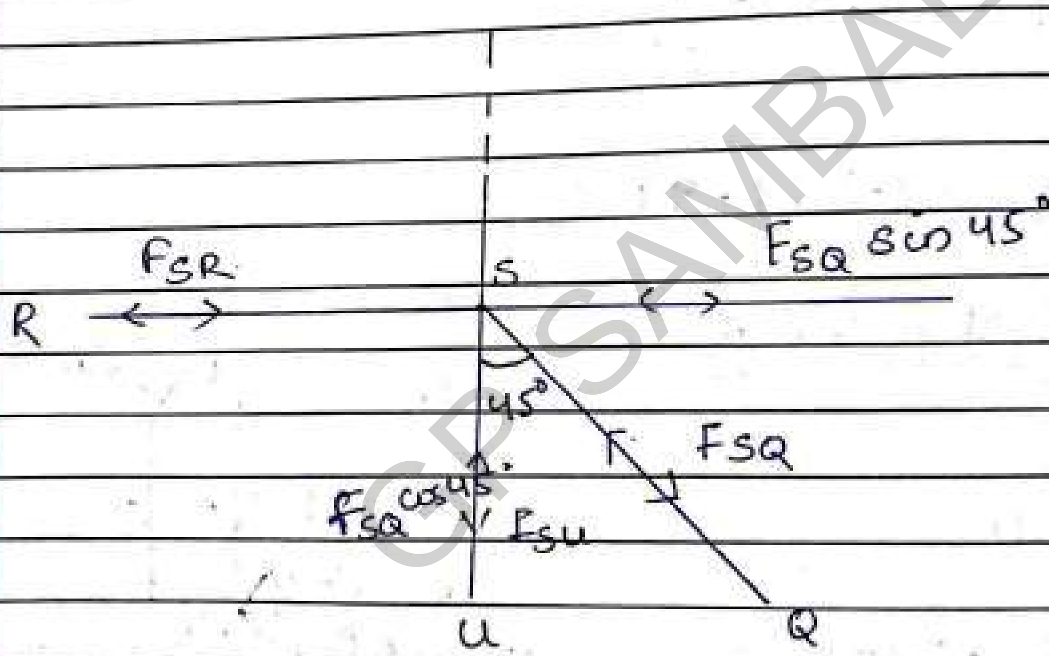
$\Sigma f_x = 0.$

$F_{QS} \cos 45 + F_{QU} = 0.$

$F_{QU} = -F_{QS} \cos 45.$

$= -56.56 \times \frac{1}{\sqrt{2}}.$

$\boxed{F_{QU} = -39.99}$ (compression)



$\Sigma f_x = 0$

$\Rightarrow F_{SR} - F_{SQ} \sin 45 = 0$

$\Rightarrow F_{SR} = F_{SQ} \sin 45°$

$= 56.56 \times \frac{1}{\sqrt{2}}.$

$\boxed{F_{SR} = 39.99 \text{ KN (Tension)}}$

Teacher's Signature

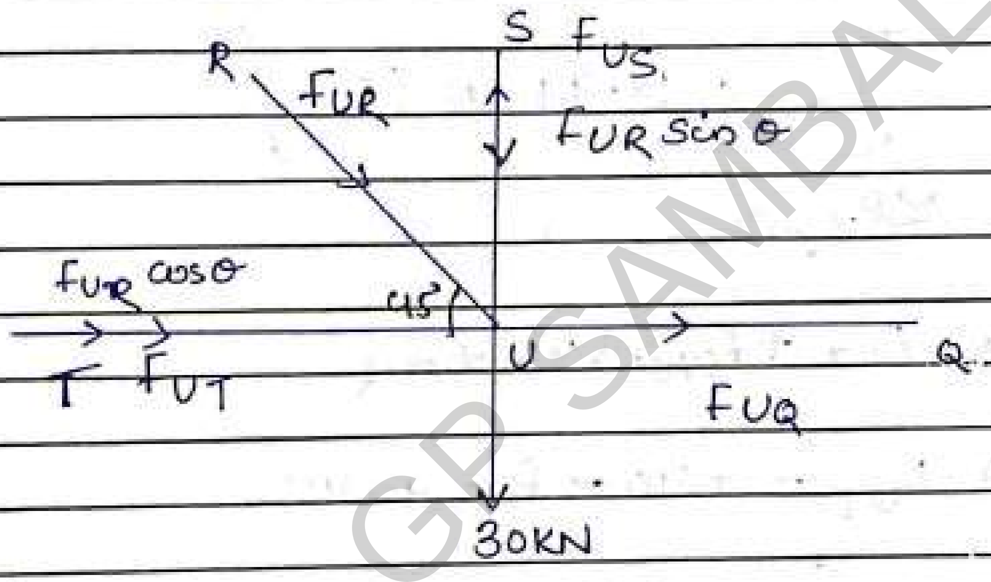
$\Sigma f_y = 0$

$F_{SU} + F_{SQ} \cos\theta = 0$

$F_{SU} = -F_{SQ} \cos 45^\circ$

$= -56.56 \times \cos 45^\circ$

$\boxed{F_{SU} = -39.99 \text{ kN}}$ (compression)



$\cancel{\Sigma f_x = 0}$

$\cancel{F_{UT} + F_{UR} \cos 45 + F_{UQ} = 0}$

$F_{UT} = \cdot$

$\Sigma f_y = 0$

$\Rightarrow f_{US} + f_{UR} \sin\theta - 30$

$\Rightarrow +30 + f_{UR} \sin 45^\circ + 40 = 0$

$\Rightarrow f_{UR} \sin 45 = -70$

$\Rightarrow f_{UR} = \dfrac{-70}{\sin 45}$

$\boxed{f_{UR} = -98.99 \text{ KN } (C)}$

$\Sigma f_x = 0$

$\Rightarrow f_{UT} + f_{UR} \cos\theta = f_{UQ}$

$\Rightarrow f_{UT} = +40 - f_{UR} \cos 45^\circ$

$= \cancel{40} (40 - (-98.99 \times \cos 45^\circ)$

$= 40 + 98.99 \times \cos 45$

$\boxed{f_{UT} = 109.99 \text{ KN } (T)}$

Find out forces in all the members with their nature as tensile or compressive as shown in fig. below [10]

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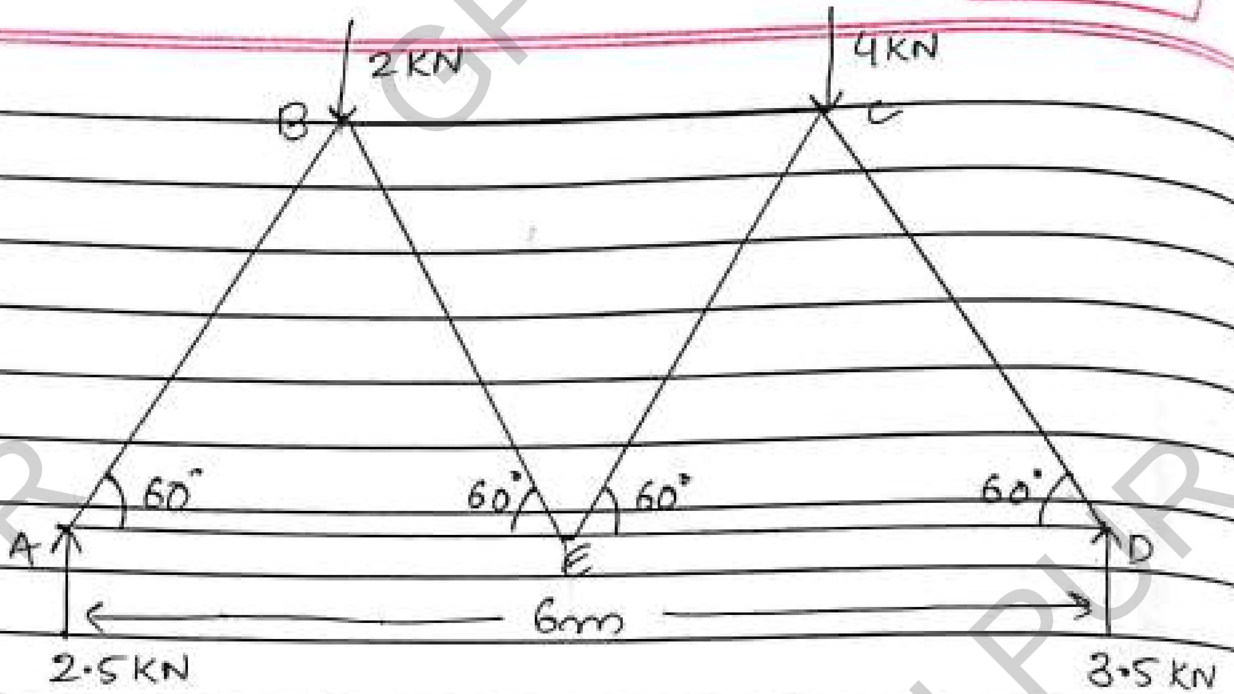


$\Sigma f_y = 0$

$R_A + R_D - 2 - 4 = 0$

$R_A + R_D - 6 = 0$

$R_A + R_D = 6\,KN$ ——— (1)

$M_A = 0$

$R_D \times 6 - 4 \times 4.5 - 2 \times 1.5 = 0$

$6R_D - 21 = 0$

$R_D = \frac{21}{6}$

$R_D = 3.5\,KN$

Teacher's Signature

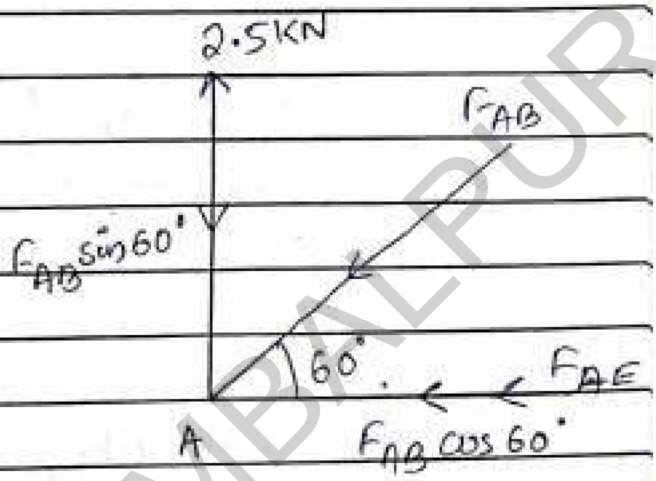
$R_A + R_D = 6\,kN$

$R_A + 3.5 = 6\,kN$

$R_A = 6 - 3.5$

$R_A = 2.5\,kN.$

For point A :-



$\Sigma f_y = 0$

$= -F_{AB}\sin 60° + 2.5 = 0$

$F_{AB}\sin 60 = 2.5$

$F_{AB} = \dfrac{2.5}{\sin 60°}$

$\boxed{F_{AB} = 2.88\,kN}$ (T)

$\Sigma f_x = 0$

$-F_{AB}\cos 60° - F_{AE} = 0$

$F_{AE} = -F_{AB}\cos 60°$

$F_{AE} = -2.88 \times \dfrac{1}{2}.$

$\boxed{F_{AE} = -1.44\,kN}$ (C)

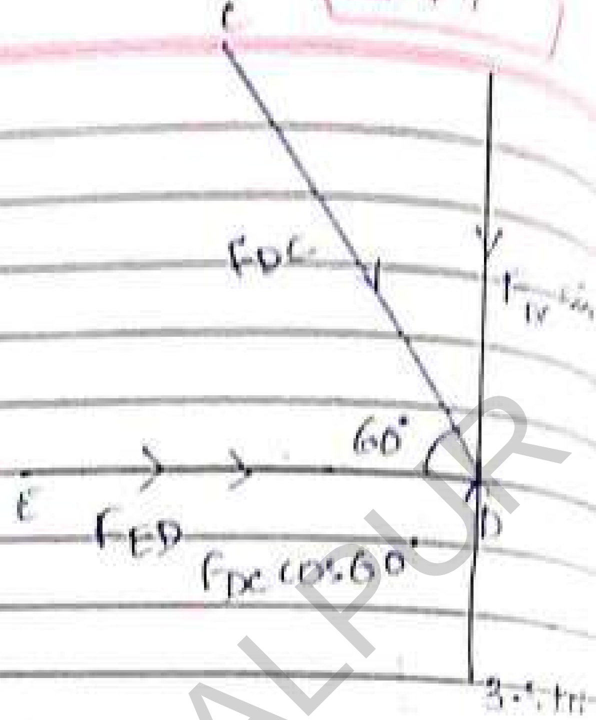
Teacher's Signature..........

$$\Sigma F_y = 0$$

$$3.5 - F_{DC} \sin 60^\circ = 0.$$

$$F_{DC} = \frac{3.5}{\sin 60^\circ}$$

$$\boxed{F_{DC} = 4.04\ kN} \ (T)$$



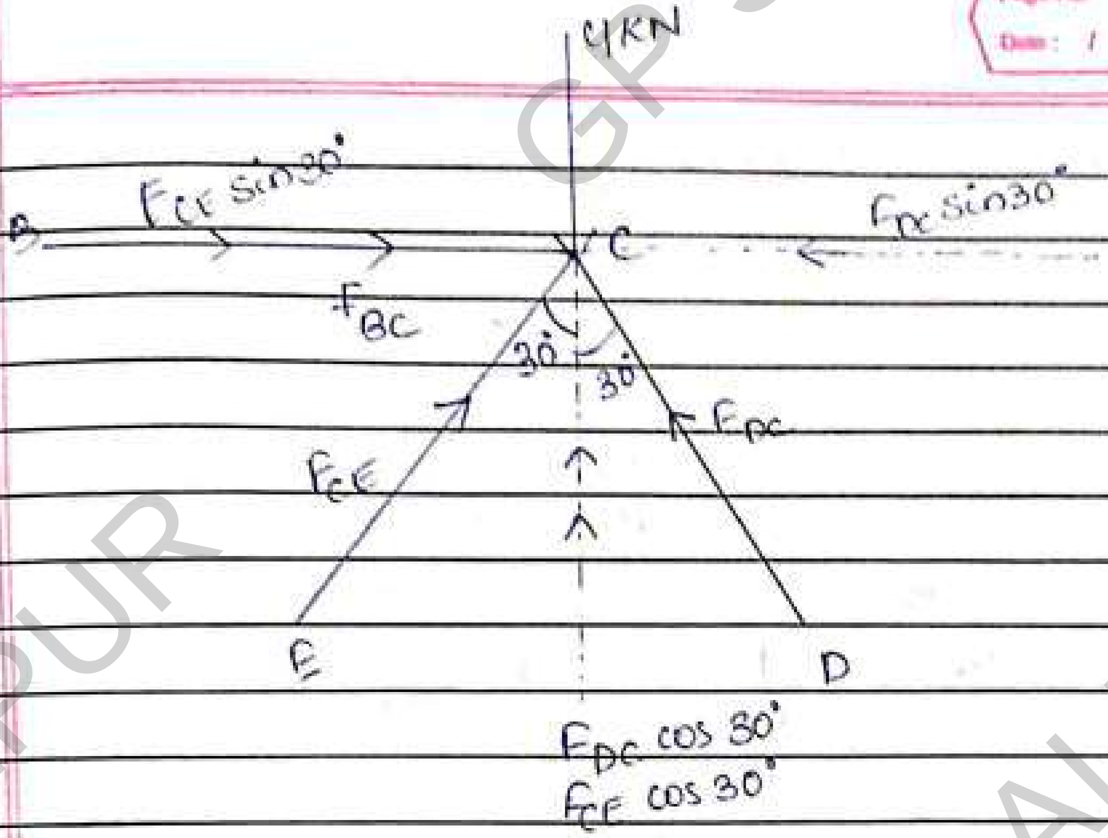
$$\Sigma F_x = 0$$

$$F_{DC} \cos 60^\circ + F_{ED} = 0$$

$$F_{ED} = -F_{DC} \cos 60$$

$$= -4.04 \times \cos 60^\circ$$

$$\boxed{F_{ED} = -2.02\ kN} \ (C)$$



$\Sigma f_y = 0$

$F_{DC}\cos 30° + F_{CE}\cos 30° - 4 = 0$

$F_{CE}\cos 30° = 4 - F_{DC}\cos 30°$

$= 4 - 4.04 \times \cos 30°$

$= 4 - 3.49$

$= \dfrac{0.51\ KN}{\cos 30°}$

$\boxed{F_{CE} = 0.588\ KN}$

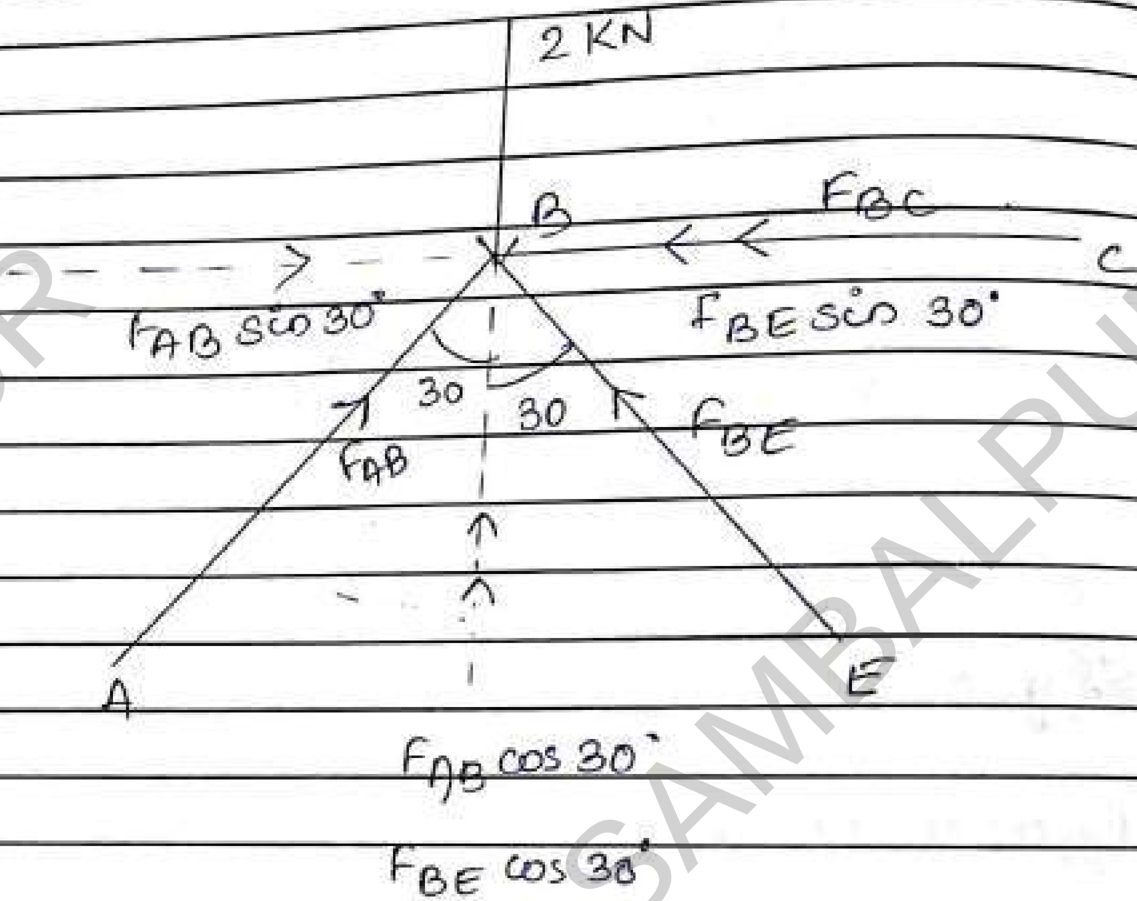
$\Sigma f_x = 0.$

$F_{BC} + F_{CE}\sin 30° - F_{DC}\sin 30° = 0.$

$F_{BC} + 0.588 \times \sin 30° - 4.04 \times \sin 30° = 0$

$F_{BC} + 0.294 - 2.02 = 0.$

$$\boxed{F_{BC} = 1.726\,KN}\ (T)$$



$\Sigma F_y = 0$

$F_{AB}\cos 30^\circ + F_{BE}\cos 30^\circ - 2 = 0$

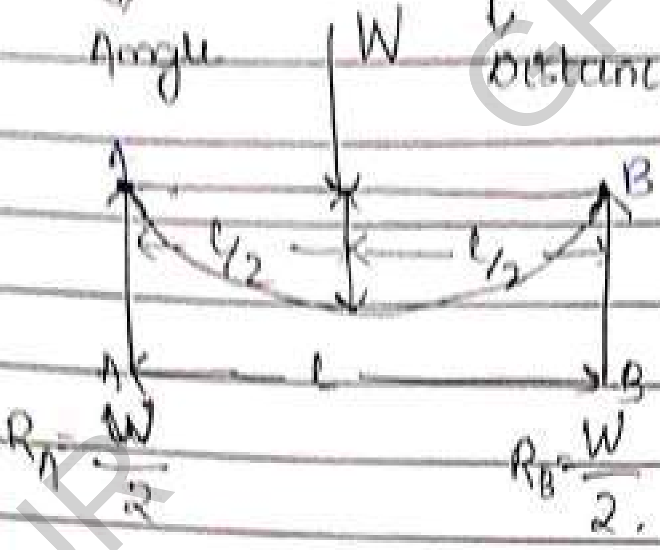
$$F_{BE}\cos 30^\circ = 2 - 2.88 \times \cos 30^\circ$$

$$= \frac{-0.49}{\cos 30}$$

$$\boxed{F_{BE} = -0.57\,KN}\ (C)$$

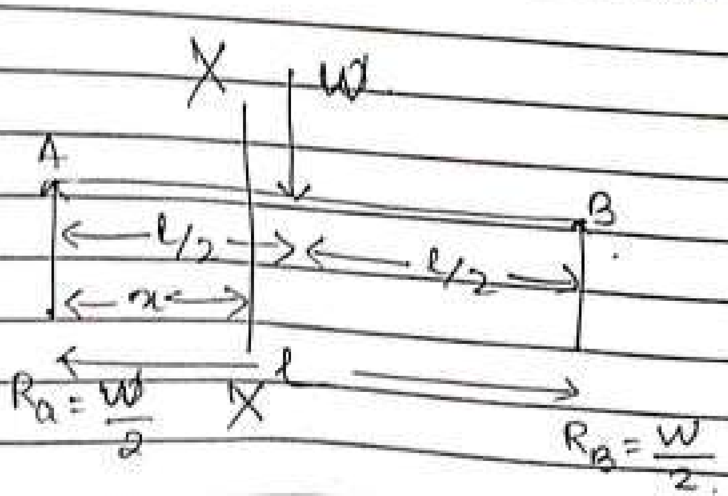
Slope and deflection :-

Angle | Distance



Slope	Deflection.
$A = \theta_A \ (x = 0)$	$A = 0 \ (x = 0)$
$B = \theta_B \ (x = l)$	$B = 0 \ (x = l)$
$C = 0$ $(x = \frac{l}{2})$ $(x = \frac{l}{2})$	$C = y_{max} \ (x = \frac{l}{2})$

Let take a section $X X'$ at a distance of 'x' from A



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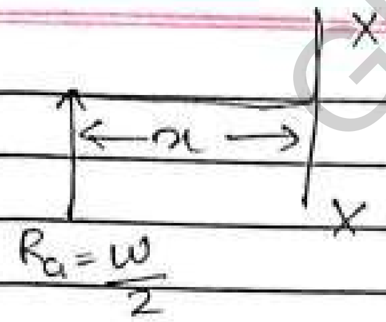


$$M_{XX} = F \times D$$

$$= \frac{W}{2} \times x$$

$$= \frac{wx}{2}$$

We know,

$$\boxed{EI \frac{d^2y}{dx^2} = M_{XX}}$$ Slope curvature Eqn.

$$EI \frac{d^2y}{dx^2} = \frac{W}{2}x \quad \text{——— (1)}$$

Integrate the Eqn -(1) w.r.t dx

$$EI \int \frac{d^2y}{dx^2} = \frac{w}{2} \int x$$

$$EI \frac{dy}{dx} = \frac{w}{2} \frac{x^2}{2}.$$

$$EI \frac{dy}{dx} = \frac{wx^2}{4} + C_1.$$

Teacher's Signature.........

$$\Rightarrow EI \frac{dy}{dx} = \frac{w}{4} x^4 + C_1$$

$$EI \frac{dy}{dx} = \frac{w}{4} x^2 + C_1$$

$$0 = \frac{w}{4} \times \left(\frac{l}{2}\right)^2 + C_1$$

$$0 = \frac{w}{4} \frac{l^2}{4} + C_1$$

$$0 = \frac{wl^2}{16} + C_1$$

$$\boxed{C_1 = -\frac{wl^2}{16}}$$

At point C

$$EI\frac{dy}{dx} = \frac{wx^2}{4} - \frac{wl^2}{16}$$

$$EI\frac{dy}{dx} = \frac{w}{4}\left(\frac{l}{2}\right)^2 - \frac{wl^2}{16}.$$

$$EI\frac{dy}{dx} = \frac{wl^2}{16} - \frac{wl^2}{16},$$

$$\boxed{\theta_C = \frac{dy}{dx} = 0.}$$

Deflection :-

(B)

$$EI \times \int \frac{dy}{dx} = \int \frac{wx^2}{4} - \frac{wl^2}{16}$$

$$EI.y = \frac{w}{4} \cdot \frac{x^3}{3} - \frac{wl^2}{16}x$$

$$EI.y = \cancel{\frac{w \cdot x^3}{4 \cdot 3}} \frac{wx^3}{12} - \frac{wl^2}{16}x + C_2$$

Section -A

$x = 0$

$$EI.y = \frac{w}{12} \times (0)^3 - \frac{wl^2}{16} \times 0 + C_2$$

$$EI.y = 0 - 0 + C_2$$

$$0 = 0 - 0 + C_2$$

$$\boxed{C_2 = 0}$$

At point (C)

$$EI\,y = \frac{w}{4}\times\frac{x^3}{3} - \frac{wl^2}{16}x$$

$$EI\,y = \frac{w}{12}\left(\frac{l}{2}\right)^3 - \frac{wl^2}{16}\times\frac{l}{2}$$

$$= \frac{w}{12}\times\frac{l^3}{8} - \frac{wl^3}{32}$$

$$= \frac{wl^3}{96} - \frac{wl^3}{32}$$

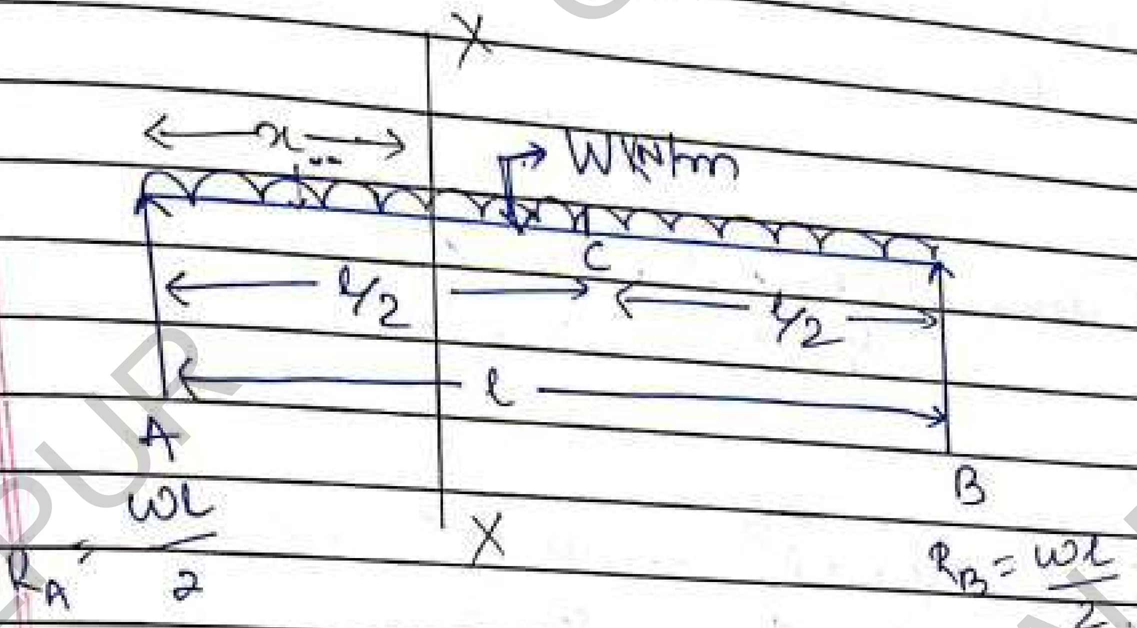
$$= \frac{wl^3 - 3wl^3}{96}$$

$$= -\frac{\cancel{2}wl^3}{\cancel{96}\,48}$$

$$\boxed{y_{max} = \frac{-wl^3}{48EI}}$$ Deflection

$$\boxed{\theta_A \Rightarrow \frac{-wl^2}{16EI} = \theta_B}$$ slope.

Simply supported beam with UDL throughout the length :-



let choose a Section XX at a distance of x m from end 'A'

$$M_{xx} = \frac{wl}{2} \times x - wx \cdot \frac{x}{2}$$

$$= \frac{wlx}{2} - \frac{wx^2}{2}$$

we know $\boxed{EI \frac{d^2y}{dx^2} = M_{xx}}$

$$EI \frac{d^2y}{dx^2} = M_{xx}$$

$$EI \int \frac{d^2y}{dx^2} = \int \frac{wl}{2} x - \int \frac{wx^2}{2}$$

$$EI \frac{dy}{dx} = \frac{wl}{2} \frac{x^2}{2} - \frac{w}{2} \frac{x^3}{3} + C_1$$

Teacher's Signature..................

$$EI\frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} + C_1$$

$$EI\,0 = \frac{wl}{4}\left(\frac{l}{2}\right)^2 - \frac{w}{6}\left(\frac{l}{2}\right)^3 + C_1$$

$$0 = \frac{wl}{4}\cdot\frac{l^2}{4} - \frac{w}{6}\frac{l^3}{8} + C_1$$

$$0 = \frac{wl^3}{16} - \frac{wl^3}{48} + C_1$$

$$0 = \frac{3wl^3 - wl^3}{48} + C_1$$

$$0 = \frac{\cancel{2}wl^3}{\cancel{48}} + C_1$$

$$C_1 = -\frac{\cancel{2}wl^3}{\cancel{48}\ 24}$$

$$\boxed{C_1 = -\frac{wl^3}{24}}$$

$$\cancel{EI.\frac{dy}{dx} = \frac{wl^3}{16} - \frac{wl^3}{48} - \frac{wl^3}{24}}$$

$$EI\frac{dy}{dx} = \cancel{\frac{wlx^2}{4}}^{\to 0} - \cancel{\frac{wx^3}{6}}^{\to 0} - \frac{wl^3}{24}$$

$$EI\frac{dy}{dx} = -\frac{wl^3}{24}$$

$$\boxed{\theta_A = \frac{-wl^3}{24EI}}$$

$$\boxed{\theta_B = \frac{-wl^3}{24EI}}$$

<u>Deflection</u>

$$EI.\int\frac{dy}{dx} = \int\frac{wlx^2}{4} - \int\frac{wx^3}{6} - \int\frac{wl^3}{24}$$

$$EI.y = \frac{wl}{4}\cdot\frac{x^3}{3} - \frac{w}{6}\frac{x^4}{4} - \frac{wl^3}{24}x + c_2$$

$$EI\,y = \frac{wl\,x^3}{12} - \frac{wx^4}{24} - \frac{wl^3x}{24} + c_2$$

Teacher's Signature..........................

$\cancel{0} = 0.$

$$\boxed{C_2 = 0}$$

$$EI.y = \frac{wl x^3}{12} - \frac{wx^4}{24} - \frac{wl^3 x}{24}$$

$$EI.y = \frac{wl(l/2)^3}{12} - \frac{w.\left(\frac{l}{2}\right)^4}{24} - \frac{wl^3\left(\frac{l}{2}\right)}{24}$$

$$EI.y = \frac{wl\,l^3}{96} - \frac{wl^4}{384} - \frac{wl^4}{48}.$$

$$EI.y = \frac{wl^4}{96} - \frac{wl^4}{384} - \frac{wl^4}{48}$$

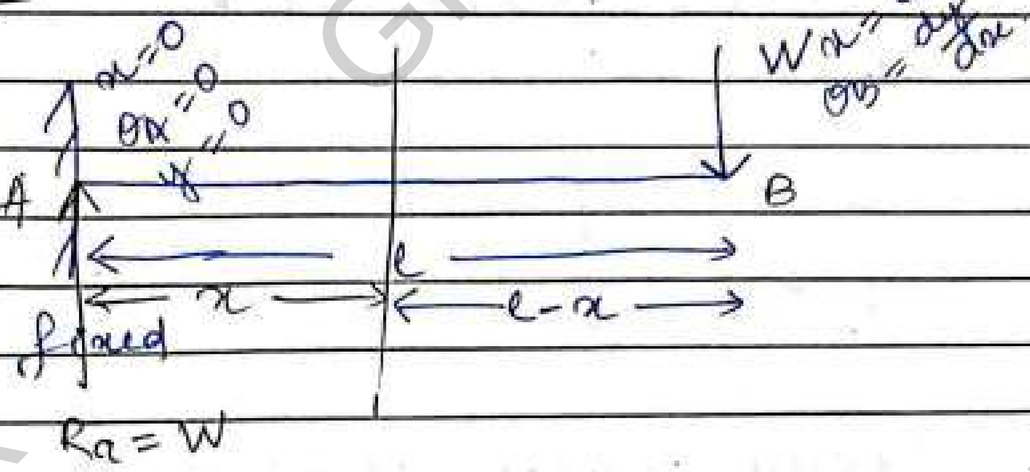
$$EI.y = \frac{4wl^4 - wl^4 - 8wl^4}{384}$$

$$EI.y = \frac{-5wl^4}{384}$$

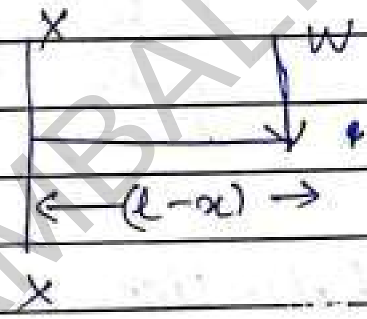
$$\boxed{y_{max} = \frac{-5wl^4}{384\,EI}}$$

Teacher's Signature ....

Cantilever beam with a point load at its free end:-



Let take a section XX from A at a distance of x



$M_{xx} = -W(l-x)$

we know

$$EI\frac{d^2y}{dx^2} = M_{xx}$$

$$EI\frac{d^2y}{dx^2} = -w(l-x)$$

$$EI\int\frac{d^2y}{dx^2} = -wl\int 1\,dx + w\int x\,dx$$

$$EI\frac{dy}{dx} = -wlx + \frac{wx^2}{2} + C_1$$

~~$EI\frac{dy}{dx}$ = ... $-\frac{wx^2}{2}$~~

Teacher's Signature...............................

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$$EI\frac{dy}{dx}\quad EI\times 0 = -wl\times 0 + \frac{w(0)^2}{2} + C_1$$

$$\boxed{C_1 = 0}$$

$EI\times\theta_B = -wl\cdot l + w\ldots$

$= \frac{-2wl^2 + wl^2}{2}$

$$\boxed{\theta_B = \frac{-wl^2}{2EI}}$$

$$EI\times\frac{dy}{dx} = wlx - \frac{wx^2}{2}$$

$$\cancel{\frac{dy}{dx} = \frac{2wlx - wx^2}{2EI}}$$

$$\Rightarrow \cancel{\frac{dy}{dx} = \frac{2wl^3 - wl^2}{2EI}}$$

<u>Deflection</u> :-

$$EI\int\frac{dy}{dx} = \int wlx\,dx - \frac{w}{2}\int x^2\,dx.$$

$$EI.y = -wl\frac{x^2}{2} + \frac{w}{2}\frac{x^3}{3} + C_2$$

$$EIy = -\frac{wlx^2}{2} + \frac{wx^3}{6} + C_2.$$

$$\cancel{EIy = \frac{wl^3}{2} - \frac{wl^3}{6} + C_2.}$$

$$\cancel{EIy =}$$

Teacher's Signature.......................

$c_2 = 0.$

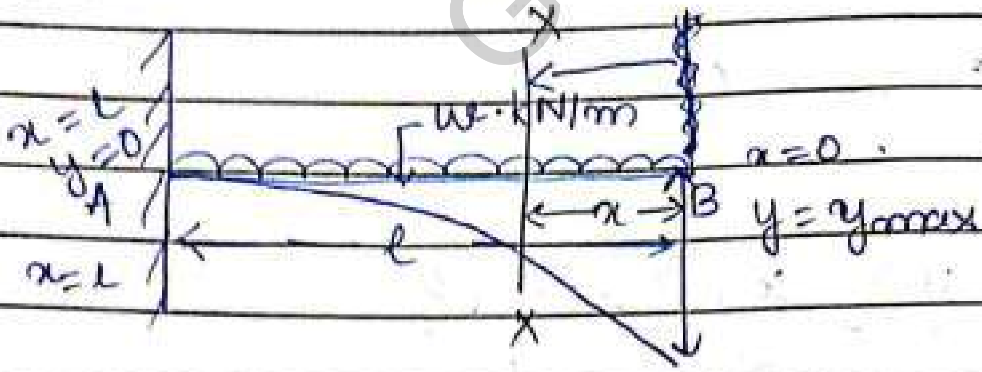
$$EI\, y = -\frac{wl^3}{2} + \frac{wl^3}{6}$$

$$EI\, y = \frac{-3wl^3 + wl^3}{6}$$

$$EI\, y = \frac{-\cancel{2}wl^3}{\cancel{6}_3}$$

$$\boxed{y_{max} = \frac{-wl^3}{3EI}}$$

Cantilever beam subjected with UDL throughout the length :-



Let take a Section XX ~~from~~ at the distance of x from end B.

$$M_{xx} = -wx \cdot \frac{x}{2}$$

$$= \frac{-wx^2}{2}$$

We know,

$$EI \frac{d^2y}{dx^2} = \frac{-wx^2}{2}$$

Integrating on both side

$$EI \int \frac{d^2y}{dx^2} = -\frac{w}{2} \int x^2$$

$$EI \frac{dy}{dx} = -\frac{w}{2} \frac{x^3}{3} + C_1$$

$$EI \frac{dy}{dx} = \frac{-wx^3}{6} + C_1$$

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~~$EI\theta_B =$~~

~~$EI \times 0 = 0 + C_1$~~ $EI \frac{dy}{dx} = -\frac{wx^3}{6} + C_1$

~~$EI \frac{dy}{dx} = \frac{-wx^3}{6}$~~ $EI \times 0 = \frac{-wl^3}{6} + C_1$

$$\boxed{C_1 = \frac{wl^3}{6}}$$

$$EI \frac{dy}{dx} = \frac{-wx^3}{6} + \frac{wl^3}{6}$$

$$EI \frac{dy}{dx} = \frac{-wx^3}{6} + \frac{wl^3}{6}.$$

At point $\frac{dy}{dx} = \theta_B$ and $x = 0$.

$\frac{dy}{dx}$ $EI \frac{dy}{dx} = 0 + \frac{wl^3}{6}.$

$$\boxed{\theta_B = \frac{wl^3}{6EI}}$$

$$\boxed{\theta_A = 0}$$

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$$EI\frac{dy}{dx} = \frac{-wx^3}{6} + \frac{wl^3}{6}.$$

Integrating both side.

$$EI\int\frac{dy}{dx} = \frac{-w}{6}\int x^3 + \frac{wl^3}{6}\int 1\,dx$$

$$EI.y = \frac{-w}{6}\frac{x^4}{4} + \frac{wl^3x}{6} + C_2.$$

$$EI.y = \frac{-wx^4}{24} + \frac{wl^3x}{6} + C_2.$$

$$\cancel{\boxed{C_2 = 0}}$$

$$\cancel{EI.y = \frac{-wx^4}{24} + \frac{wl^3x}{6}.}$$

$$\cancel{EI\,y_{max} = \frac{-wx^4}{24}}$$

$$0 = \frac{-wl^4}{24} + \frac{wl^4}{6} + C_2.$$

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$$0 = \frac{-wl^4 + 4wl^4}{24} + C_2$$

$$0 = \frac{3wl^4}{24} + C_2$$

$$C_2 = -\frac{\cancel{3}wl^4}{\cancel{24}_{8}}$$

$$\boxed{C_2 = -\frac{wl^4}{8}}$$

$$EI \cdot y = -\frac{wx^4}{24} + \frac{wl^3x}{6} - \frac{wl^4}{8}$$

$$\boxed{y_{max} = -\frac{wl^4}{8EI}}$$

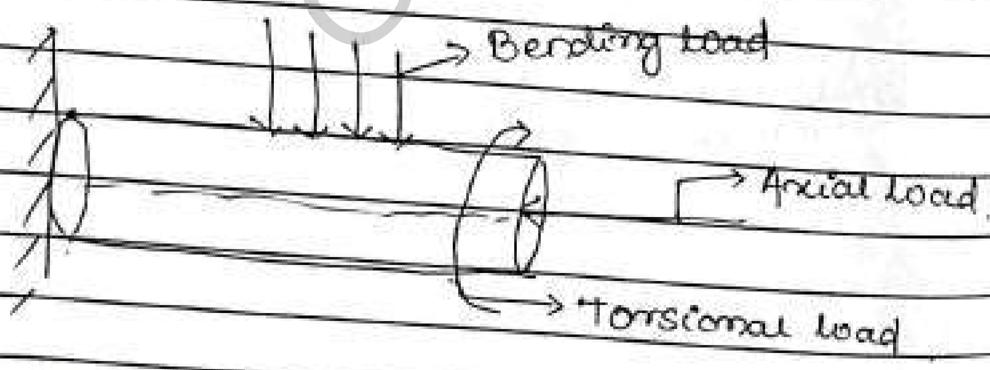
Stresses in Shaft due to torsion :-

Shaft - The shafts are usually cylinderical in Section & Solid or hollow.

- They are made of mild steel and copper alloy.

- Shaft may be subjected to following loads :-

(a) Torsional load
(b) Bending load
(c) Axial load
(d) combination of the above three.



- Shaft is a member like tunnel, pipe for pumping water, conveyance for the men or Material.

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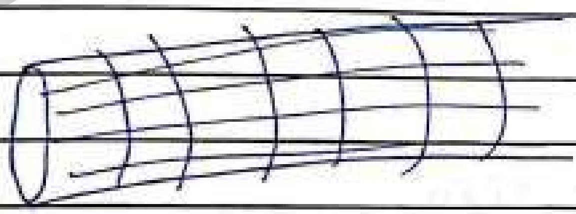
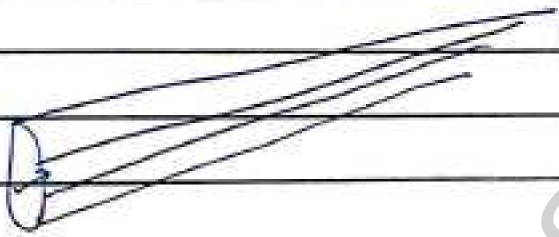
Torsion :-

- Torsion is a moment that twist or deforms a member about its longitudinal axis.

- By observation, if angle of rotation is small, length of the shaft and its longitudinal axis remain unchanged.



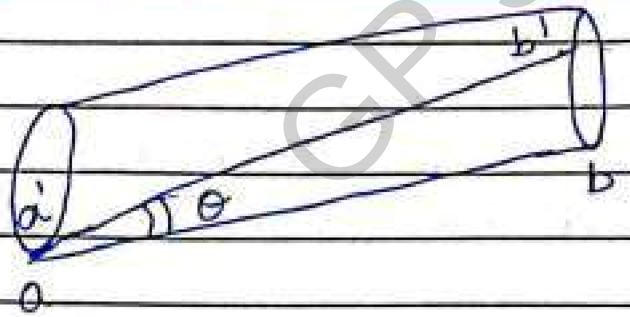


After Deformation,

- Only ~~Lo~~ longitudinal lines becomes twisted and remain unchanged

Shear Strain due to torsion :-

The angular displacement of the one surface of the shaft from its original position due to the applied of torque.



- θ can be measured radian between the final and original position of the bar.

$$180^\circ = \pi$$

$$\boxed{1^\circ = \frac{\pi}{180^\circ}}\text{ radian}$$

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Torsional Equation for Shaft :-

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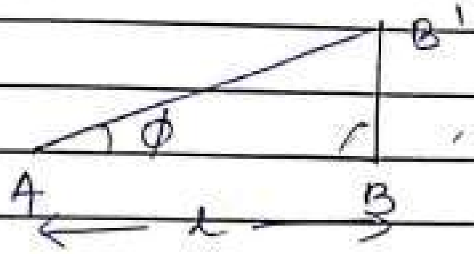
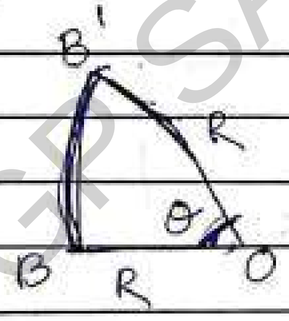
θ = angle of twist
φ = angle of shear stress



$$\tan\phi = \frac{BB'}{L}$$



Length of Arc BB'
$= R \times \theta$

Putting the value of Arc in tan φ

$$\cancel{\tan\phi}$$

$$\tan\theta = \frac{R\theta}{L}$$

$$\tan\phi = \frac{BB'}{L} = \frac{R\theta}{L}$$

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ϕ is very very small, ~~tan~~

i.e, $\boxed{\tan\phi = \phi}$

$$\boxed{\phi = \frac{R\times\theta}{l}} \quad \text{——— } (1)$$

$$G = \frac{\text{Shear stress}}{\text{shear strain}} = \frac{\tau}{\phi}$$

$$G = \frac{\tau}{\phi}$$

$$G\times\phi = \tau$$

$$\boxed{\phi = \frac{\tau}{G}} \quad \text{——— } (2)$$

from (1) and (2)

$$\boxed{\frac{R\times\theta}{L} = \frac{\tau}{G}}$$

from eqn -(1) and (2) we get that

$$\boxed{\frac{R\times\theta}{L} = \frac{\tau}{G}}$$

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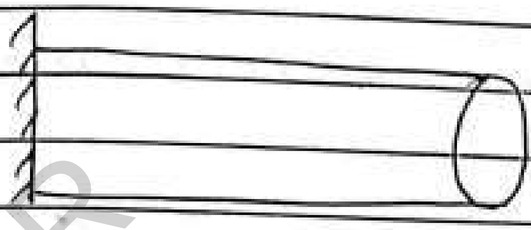
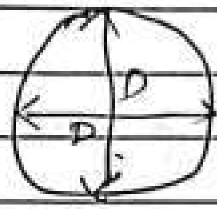
$$\boxed{\frac{G\theta}{l} = \frac{\tau}{R}}$$



We know that MI of a circle $= \frac{\pi D^4}{64}$.



Polar moment of inertia :-

$$I_p = I_{xx} + I_{yy}$$

$$= \frac{\pi D^4}{64} + \frac{\pi D^4}{64}$$

$$= \frac{\pi D^4 + \pi D^4}{64}$$

$$= \frac{\cancel{2}\pi D^4}{\cancel{64}\,32}$$

$$\boxed{I_p = \frac{\pi D^4}{32}}$$

It is denoted by I_p or J

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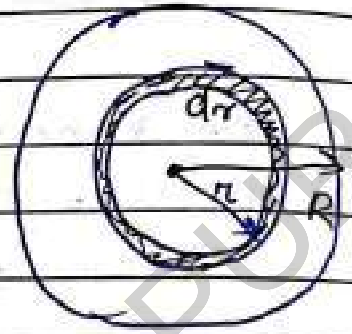
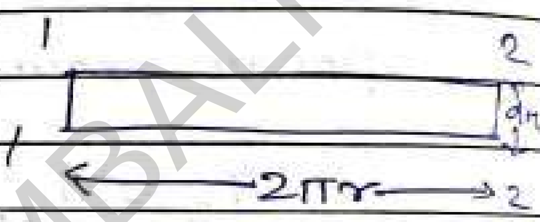
$$\frac{\tau}{R} = \text{constant}$$

Lets choose a strip from the circular section
thickness = dr



Area of the small strip
$= L \times \text{Thickness}$
$= 2\pi r \times dr.$



$$\tau_1 = \frac{F}{A}$$

$$\tau_1 = \frac{F}{2\pi r \times dr}$$

$$\frac{\tau}{R} = \text{constant}$$

$$\boxed{\frac{\tau}{R} = \frac{\tau_1}{r}}$$

$$\tau_1 = \frac{\tau}{R} \cdot r$$

$$\frac{\tau}{R} \times r = \frac{F}{2\pi r \times dr}$$

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$$\frac{\tau}{R} \times r \times 2\pi r \times dr = F$$

Torque = Force x Radial distance

$$T = F \times r$$

$$T = \frac{\tau}{R} \times r \times 2\pi r \times dr \times r$$

$$T = \frac{\tau}{R} \times r^3 \times 2\pi \times dr$$

$$T = \int_0^R \frac{\tau}{R} \times r^3 \times 2\pi \times dr$$

$$T = \frac{\tau 2\pi}{R} \int r^3 \, dr$$

$$\frac{2\pi\tau}{R} \left[\frac{r^4}{4} \right]_0^R + C$$

$$= \frac{2\pi\tau}{R} \left[\frac{R^4}{4} - \frac{0^4}{4} \right] + C$$

$$T = \frac{2\pi\tau}{\cancel{R}} \frac{R^{\cancel{4}3}}{4} + C$$

$$T = \frac{\cancel{2}\pi\tau R^3}{\cancel{4}2} \Rightarrow T = \frac{\pi\tau R^3}{2}$$

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$$T = \frac{\tau\pi}{2} \times R^3 \cancel{\times \frac{2\pi}{2}}$$

$$= \frac{\tau\pi}{2} \times \left(\frac{D}{2}\right)^3 \cancel{\times \frac{2\pi}{2}}$$

$$\boxed{T = \frac{\tau\pi D^3}{16}} \cancel{\times \frac{2\pi}{2}}$$

$$\Rightarrow \tau = \frac{16T}{\pi D^3}$$

$$\frac{G\theta}{l} = \frac{\tau}{R}$$

$$\frac{G\theta}{l} = \tau \times \frac{1}{R}$$

$$\frac{G\theta}{l} = \frac{16T}{\pi D^3} \times \frac{1}{R}$$

$$\frac{G\theta}{l} = \frac{16T}{\pi D^3 R}$$

$$\frac{G\theta}{l} = \frac{16T}{\pi D^3 \times \frac{D}{2}}$$

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$$\frac{G\theta}{l} = \frac{16T}{\pi D^3 \times D} \times 2$$

$$\frac{G\theta}{l} = \frac{32T}{\pi D^4}$$

$$\frac{G\theta}{l} = \frac{T}{I_p}$$

$$\boxed{\frac{T}{I_p} = \frac{G\theta}{l} = \frac{\tau}{R}}$$

***

Numericals :-

A solid steel shaft 5m long is stretched at 80mpa when twisted through 4°. use G = 83 GPa compute the shaft diameter :-

$l = 5m$

$G = 83 \times 10^9 \, N/m^2$.

$\tau = 80 \times 10^6 \, N/m$

$\theta = 4°$

$= \frac{4 \times \pi}{180}$

$= 0.02\pi$

$$\frac{G\theta}{l} = \frac{\tau}{R}$$

$$R = \frac{\tau l}{G\theta}$$

$$= \frac{80 \times 10^6 \times 5}{83 \times 10^9 \times 0.02\pi}$$

$\cancel{0.076m}$ $= \frac{80 \times 10^6 \times 5}{83 \times 10^9 \times 0.07}$

$\cancel{D = 2 \times 0.076}$

$\cancel{= 0.152}$

$R = 0.069$

$D = 2 \times 0.069$

$= 0.138m$

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Bending Stress

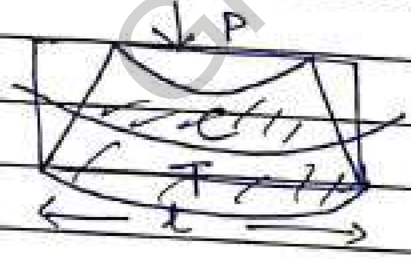
When some external load on a beam, the shear force and bending moments are set up at all section of the beam.

Due to shear force and bending moment, the beam undergoes certain deformation the material of the beam will offer resistance or stress against these deformation.

The stress introduced by the bending moment are known as bending stress.

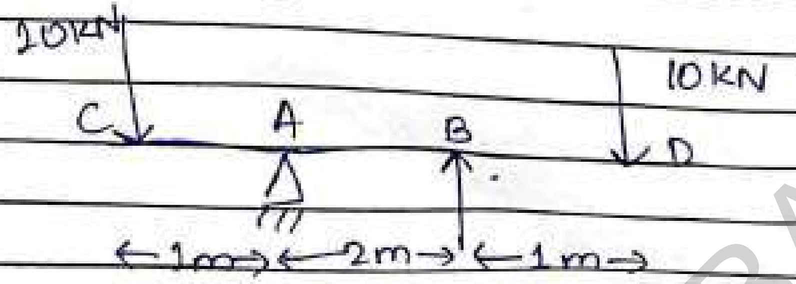
Neutral axis :- (NA)

↳ Neutral axis is the line of intersection which divides the beam or member into two zone i.e, compression zone and tension zone



↳ Neutral axis remain unchanged

Pure Bending



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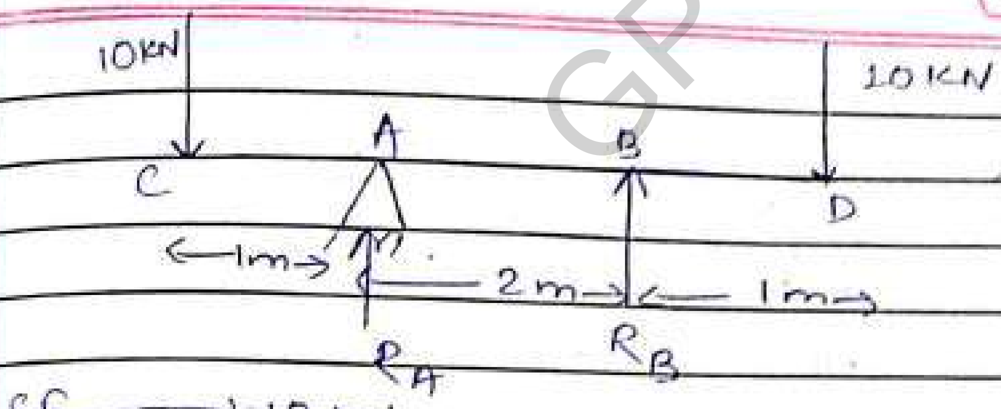


~~$SF_D = +10$ kN~~

$R_A + R_B - 10 - 10 = 0$

$R_A + R_B - 20 = 0$

$R_A + R_B = 20$ ——— (1)

$M_A = 0$

$-10 \times 3 + R_B \times 2 - 10 \times 1 = 0$

$-30 + 2R_B - 10 = 0$

$2R_B - 40 = 0$

$R_B = \frac{40}{2} = 20$

$R_B = 20$ kN

$R_A + R_B = 20$.

$R_A = 20 - 20$

$R_A = 0$

$BM_D = 0$ kN/m

$BM_B = -10 \times 1 = -10$ kN/m

$BM_A = -10 \times 3 + 20 \times 2 + 0$

$= -30 + 40 = 10$ kN/m

$BM_C = -10 \times 4 + 20 \times 3 + 0 \times 1 + 10 \times 0$

$= -40 + 60$

$= 20$ kN/m.

$SF_D = +10$ kN

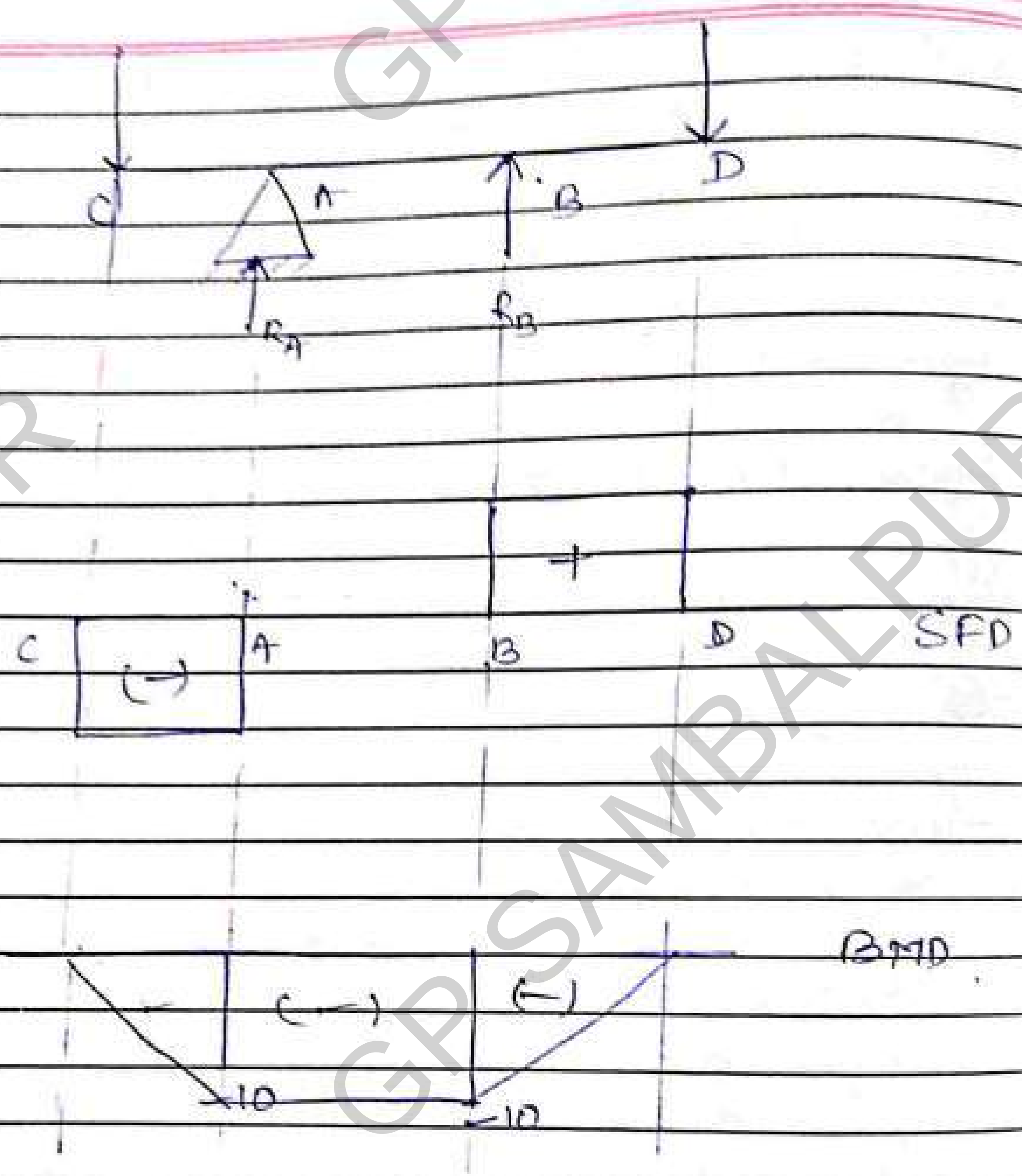
$SF_B = -20 + 20 = 10$ kN

$SF_A = 10$ kN

$SF_C = 0 + 10 - 20 - 0 = 0$

$+10 - 10 - 10$

-10 kN



Simple bending 1. SF = 0
2. BM = constant