

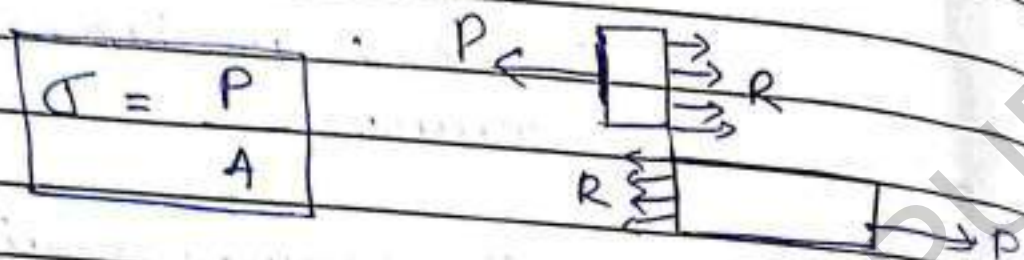
2 Marks

Simple Stresses and Strains

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Stress - It is the resistance offered by a body against the deformation is known as stress.



It is denoted by the Symbol Sigma (σ)

where P = Force or Load

A = Cross-sectional area

unit of Stress - N/m^2

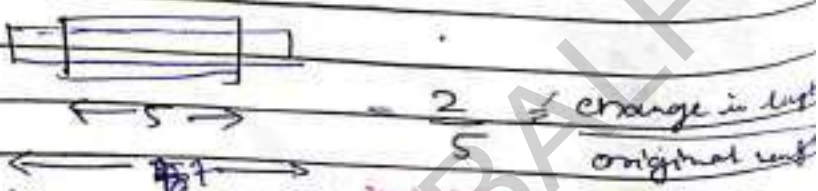
Strains :- It is the ratio of change in dimension to original dimension

- It is denoted by Symbol 'e'

$$e = \frac{\Delta l}{l}$$

→ change in length
→ original length.

- It is a unitless quantity



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Types of Strains :-

- (i) Longitudinal Strain
- (ii) Lateral Strain
- (iii) Volumetric Strain

(i) Longitudinal Strain :-

$$e = \frac{\Delta L}{L}$$

(ii) Lateral Strain :-

$$e = \frac{\Delta d}{d}$$

(iii) Volumetric Strain :-

$$e = \frac{\Delta V}{V}$$

Hooke's law :- [2 Marks]

Statement :-

It states that "when a material is loaded such that the ratio of intensity of stress to the strain is a constant"

Mathematically

$$\frac{\sigma}{e} = \text{constant}$$

$$\sigma \propto e$$

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{e}$$

E = young's modulus of elasticity

$$\text{unit} = \text{N/m}^2$$

A load of 5 kN is to be applied to develop 100 MPa stress. Find the diameter of the wire.

$$\sigma = \frac{P}{A}$$

$$100 \times 10^6 \frac{\text{N}}{\text{m}^2}$$

$$P = 5 \text{ kN} = 5 \times 1000 = 5000 \text{ N}$$

$$A = \frac{P}{\sigma}$$

$$= \frac{5000}{100 \times 10^6}$$

$$= 5 \times 10^{-5} \text{ m}^2$$

$$A = \frac{\pi \times D^2}{4}$$

$$D^2 = \frac{4 \times 5 \times 10^{-5}}{3.14}$$

$$= 7.97 \times 10^{-3} \times 1000 = 7.97 \text{ mm}$$

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$$\sigma \propto e$$

$$\frac{P}{A} \propto \frac{\Delta L}{L}$$

$$\frac{P}{A} = C \frac{\Delta L}{L}$$

$$\frac{P}{A} = \frac{E \times \Delta L}{L}$$

$$\Delta L = \frac{P L}{E A}$$

Q1 A Steel rod of ~~500~~ 500 mm long and 20 mm x 10 mm in cross-section is subjected to axial load of 300 kN. If the modulus of Elasticity $E = 2 \times 10^5 \text{ N/m}^2$, calculate the Elongation and Strain.

$$L = 500 \times 10^{-3} \text{ m}$$

$$E = 2 \times 10^5 \text{ N/m}^2$$

$$P = 300 \times 10^3 \text{ N}$$

$$A = 20 \times 10 = 200 \text{ mm}^2$$

$$\sigma = 3.75$$

$$500 \times 10^{-3}$$

$$= 7.5 \times 10^{-3}$$

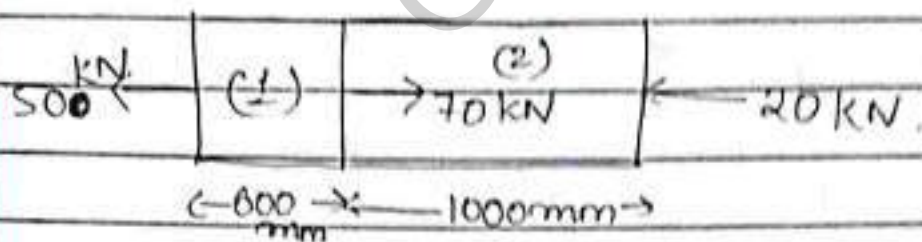
$$A = 20 \times 10^{-3} \times 10 \times 10^{-3} \text{ m}^2$$

$$= 2 \times 10^{-4} \text{ m}^2$$

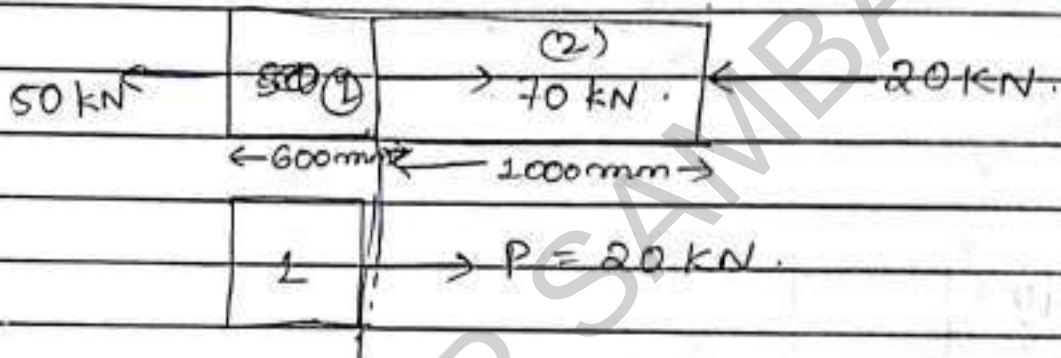
$$\Delta L = \frac{300 \times 10^3 \times 500 \times 10^{-3}}{2 \times 10^5 \times 2 \times 10^{-4}}$$

$$\Delta L = 3.75 \times 10^{-3} \text{ m}$$

$$= 3.75 \text{ mm}$$

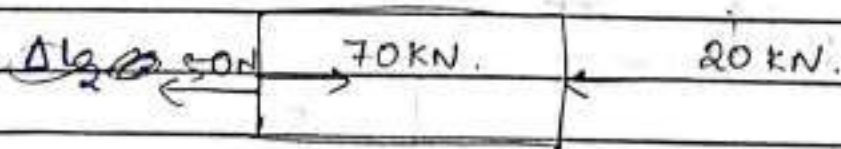


A Bar having cross-sectional area of 1000 mm^2 is subjected to an axial force given in the figure. Find the change in length of the bar take $E = 1.05 \times 10^5 \text{ N/mm}^2$.



$$\Delta l_1 = \frac{50 \times 10^3 \times 0.6}{1.05 \times 10^5 \times 10^{-3}}$$

$$= 0.114 \text{ m} \quad 0.285 \text{ m} \quad 285.71$$



$$\Delta l_2 = \frac{20 \times 10^3 \times 1}{1.05 \times 10^5 \times 10^{-3}}$$

$$= 0.190 \text{ m} \quad 95.234 \text{ m}$$

$$\Delta l = 0.114 + 0.190 = 0.304 \text{ m}$$

$$\Delta L_1 = \frac{50 \times 10^3 \times 600}{1.05 \times 10^5 \times 1000}$$

$$= 0.285 \text{ mm}$$

$$\Delta L_2 = \frac{20 \times 10^3 \times 1000}{1.05 \times 10^5 \times 1000}$$

$$= 0.190$$

$$\Delta L = 0.095 \text{ mm}$$

Shear Stress Distribution in Rectangular cross-section

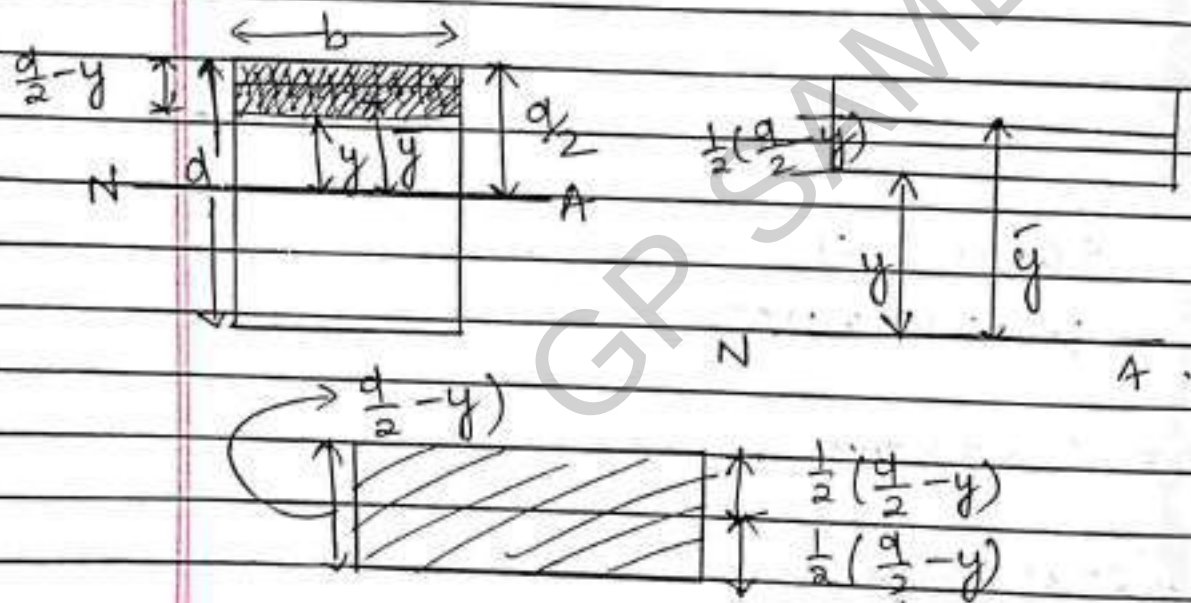
[5 - 10]
(N) (T4N)

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Shear Stress :-

- Force acting parallel to the cross-sectional area
- It causes resistance to the shearing member
- Direction is opposite to the applied load
- It is denoted by symbol (τ)



$$\text{Area} = b \times \left(\frac{d}{2} - y \right)$$

$$I = \frac{b d^3}{12}$$

$$\bar{y} = \frac{1}{2} \left(\frac{d}{2} - y \right)$$

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b = width of rectangle

d = depth of rectangle

NA = Neutral axis

A = Area of Strip

$\bar{y}$ = CG of the Strip to the neutral axis

y = Distance of the Strip to the neutral axis

I = Moment of inertia

$$\tau = F \times \frac{A \bar{y}}{I \cdot b}$$

$$= F \times b \times \left(\frac{d}{2} - y \right) \times \left\{ y + \frac{1}{2} \left(\frac{d}{2} - y \right) \right\}$$

$$\frac{bd^3}{12} \times b$$

At the NA

$$y = 0$$

$$\tau = F \times \frac{bd}{2} \times \frac{d}{2 \cdot 4}$$

$$\frac{bd^3}{12} \cdot b$$

$$= F \times \frac{bd^2}{8}$$

$$\frac{b^2 d^3}{12}$$

$$= F \times \frac{bd^2}{8} \times \frac{12}{b^2 d^3} = \frac{3F}{2bd} = \frac{3}{2} \frac{F}{bd}$$

$$\tau = \frac{3}{2} \frac{F}{bd}$$

At outermost fiber

$$y = \frac{d}{2}$$

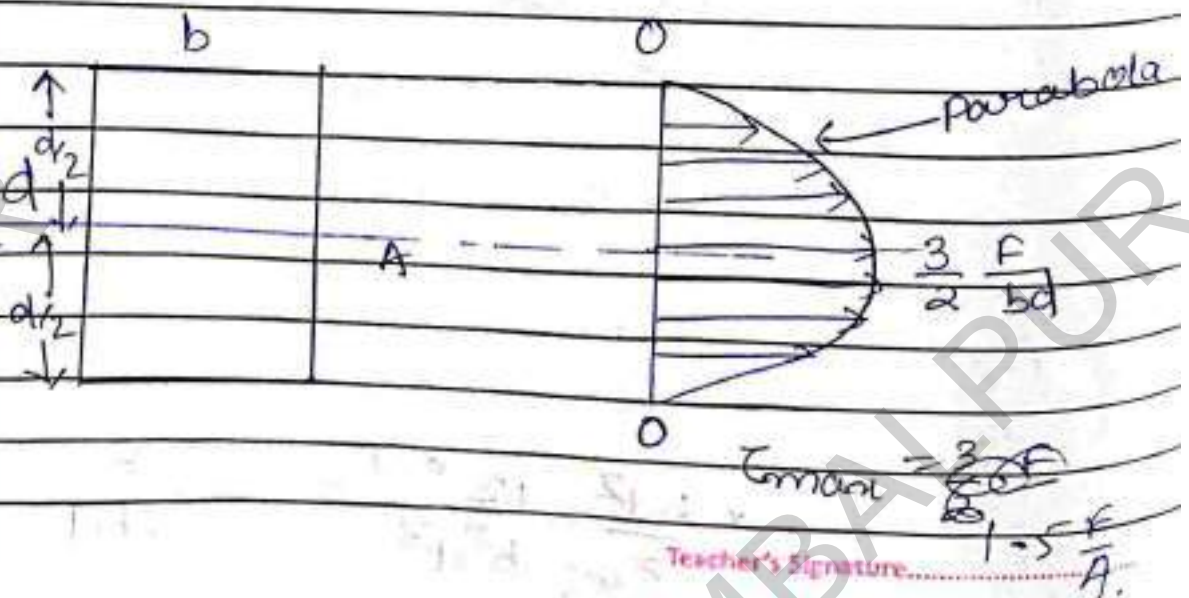
$$\tau = F \times b \times \left(\frac{d}{2} - y \right) \times \left(y + \frac{1}{2} \left(\frac{d}{2} - y \right) \right)$$

$$\frac{bd^3}{12} \times b$$

$$\tau = F \times b \times \left(\frac{d}{2} - \frac{d}{2} \right) \times \left(\frac{d}{2} + \frac{1}{2} \left(\frac{d}{2} - \frac{d}{2} \right) \right)$$

$$\frac{bd^3}{12} \times b$$

$$\tau = F \times 0 = 0$$

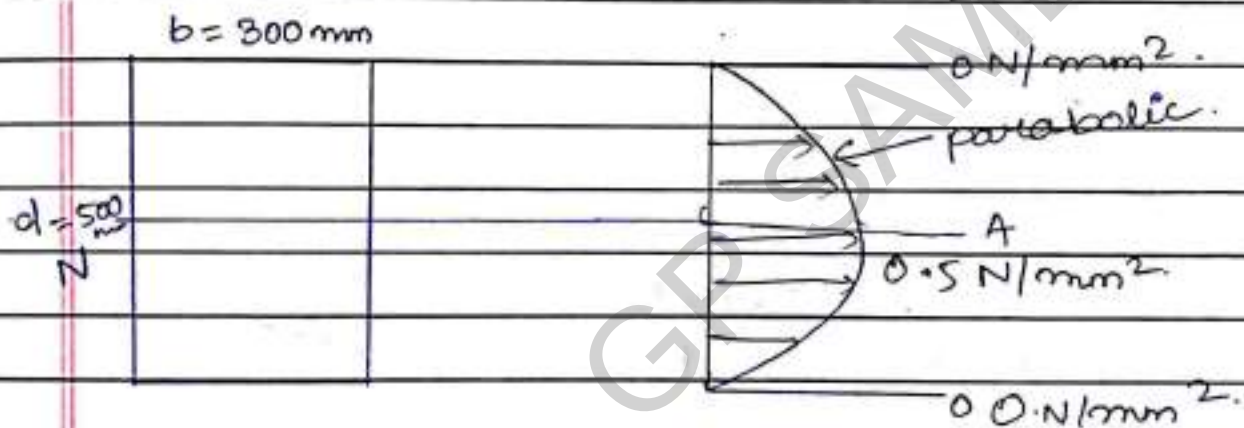


Q) A rectangular beam of cross-section is $300\text{mm} \times 500\text{mm}$ is subjected by to a maximum shear force of 50 kN find the maximum shear stress and draw the distribution.

$$\tau = \frac{3}{2} \times \frac{50 \times 10^3}{300 \times 500}$$

$$= \frac{1}{2}$$

$$= 0.5 \text{ N/mm}^2.$$



Circular beam :-

$$\tau_{\text{max}} = \frac{4}{3} \frac{F}{A}$$

$$\tau_{\text{max}} = 1.33 \frac{F}{A}$$

A rectangular beam 10cm wide is subjected to the maximum shear force of 50 kN, the corresponding maximum shear stress being 3 N/mm^2 . Calculate the depth of beam

$$\rightarrow \text{width } (b) = 10 \text{ cm} \\ = 100 \text{ mm}$$

$$\text{Shear force } F = 50 \text{ kN}$$

$$\text{Shear stress } \tau_{\text{max}} = 3 \text{ N/mm}^2$$

$$d = ?$$

$$\tau = \frac{3}{2} \frac{F}{bd}$$

$$3 = \frac{3}{2} \times \frac{50 \times 10^3}{100 \times d}$$

$$3 = \frac{1.5 \times 50 \times 10^3}{100 \times d}$$

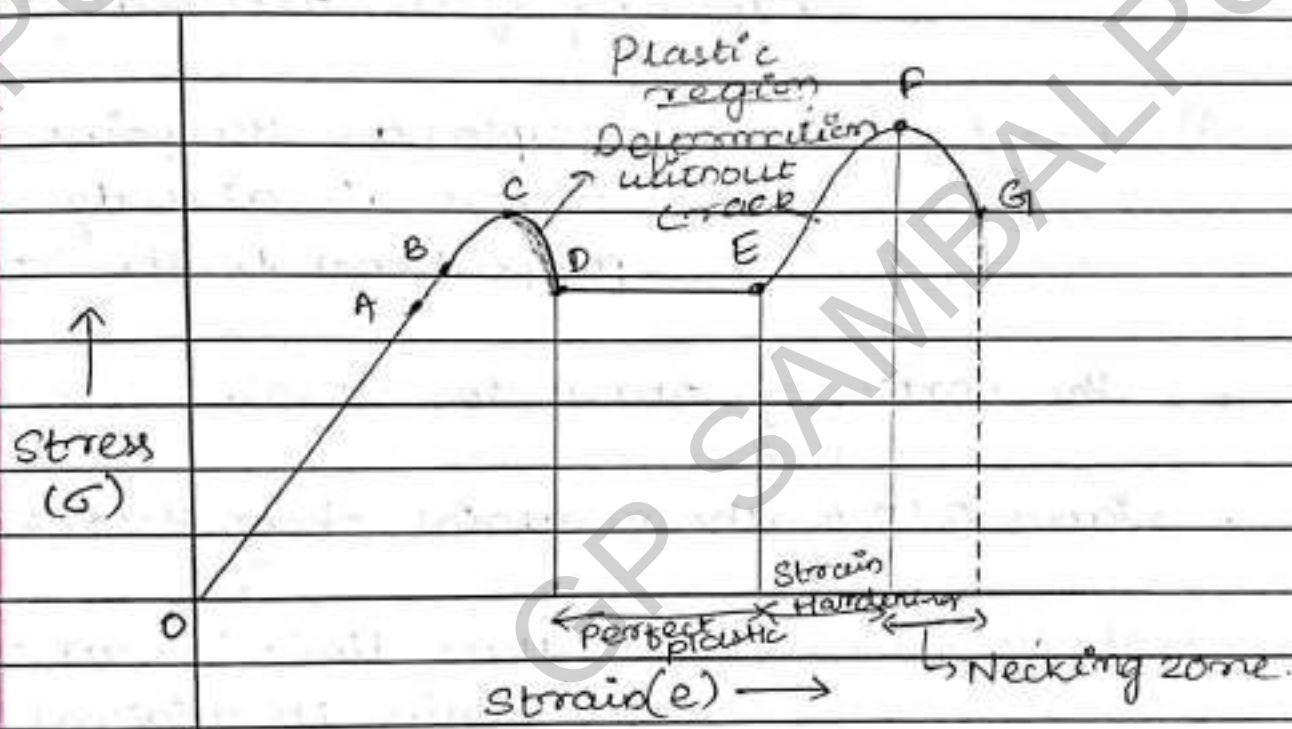
$$d = \frac{1.5 \times 50 \times 10^3}{100 \times 3}$$

$$d = 250 \text{ mm}$$

Stress - Strain curve for mild steel bar.

When a ductile material like mild steel is subjected to tensile force, it undergoes different stages.

Stress-strain curve is the graphical representation of these stages.



A → proportional limit

B → Elastic limit

C → upper yield point

D → lower yield point

E → Strain Hardening

F → ultimate stress point

G → failure point or breaking point

Elasticity:- It is the property of a material which allows to return its original shape after removing the external force.

Plasticity:- It is a property of a material which does not allow to return its original shape even after removing of the external force.

(A) Proportional limit:- upto the ~~the~~ point A the stress is directly proportional to the strain.

- The ratio of stress to strain is constant.

- From 0 to A the material obeys Hooke's law.

(B) Elastic limit:- upto this limit (B) material will regain its original shape after removing the external load.

(C) & (D) Upper yield point and lower yield point:-

The upper yield point is the stress level just before yielding starts.

The lower yield point is the stress level at which yield maintained.

(E) Strain Hardening :-

- In these point the material starts offering resistance against deformation
- This is because of change in the structure of mild steel.

(F) Ultimate Stress :- (Point F)

- This is the maximum stress that a material can bear

(G) Failure point :- (Point G)

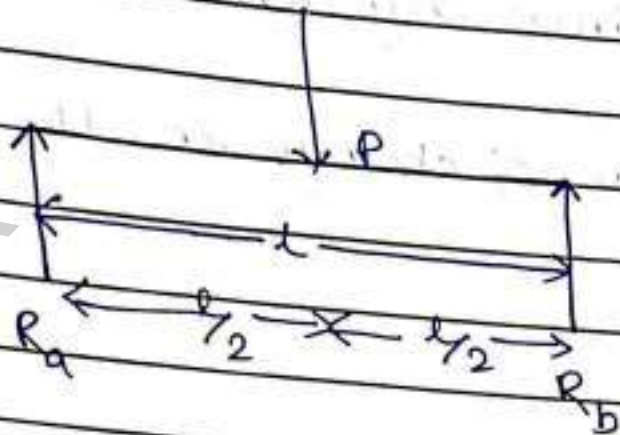
- In these point the stress & strain curve where material fails is known as Breaking point
- Stress corresponding to these point is known as failure stress or Breaking stress

Shear Force and Bending moment Diagram

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Simple Supported beam with point load at mid section.



$$\sum F_y = 0$$

$$R_a + R_b - P = 0$$

$$\sum M_A = 0$$

$$-P \times \frac{l}{2} + R_b \times l = 0$$

$$R_b \times l - P \times \frac{l}{2} + R_a \times 0 = 0$$

$$R_b \times l - \frac{Pl}{2} = 0$$

$$R_b \times l = P \times \frac{l}{2}$$

$$R_b \times l = \frac{Pl}{2}$$

$$R_b = \frac{Pl}{2l}$$

$$R_b = \frac{P}{2}$$

UDL

uniformly distributed Load



uniformly Varying Load

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$$R_a + R_b - P = 0.$$

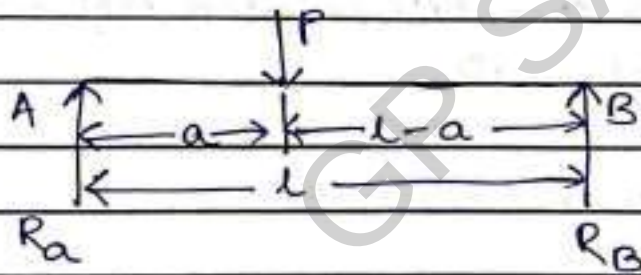
$$R_a + R_b = P$$

$$R_a + \frac{P}{2} = P$$

$$R_a = P - \frac{P}{2}$$

$$R_a = \frac{P}{2}$$

Simply Supported with a point load at a distance 'a' from the left hand support:-



$$\sum f_y = 0.$$

$$R_a + R_b - P = 0$$

$$\sum M_A = 0$$

$$R_b \times l - P \times a = 0.$$

$$R_b = \frac{Pa}{l}$$

$$R_a + R_b - P = 0$$

$$R_a = P - R_b$$

$$R_a = P - \frac{Pa}{l}$$

$$R_a = \frac{1P - Pa}{l}$$

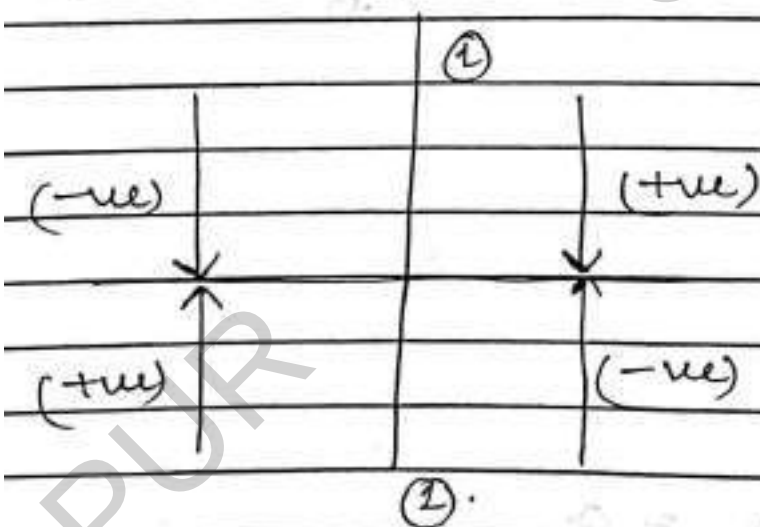
$$R_a = \frac{P(l-a)}{l}$$

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Shear Force

Shear force may be defined as the net sum of all the forces including the reaction acting at that section either from left side or right side of that section.

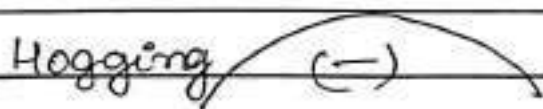
Sign convention



Bending ~~Force~~ ^{Moment} :-

Bending moment at a section may be defined as the net sum of all the moment either from left side or from right side.

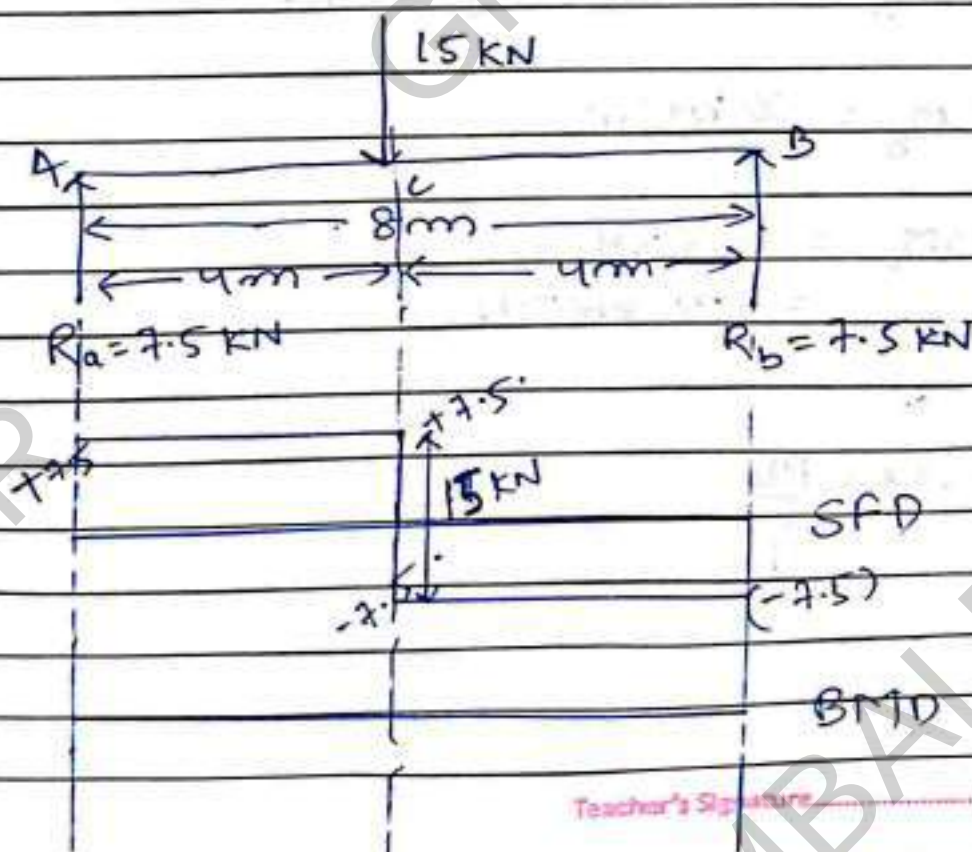
Sign convention



ACW - +ve

CW - -ve

A simply supported beam of 8m span length, having a ~~tee~~ point load of 15 kN at the centre of the beam. Draw the SFD and BMD.



when load is applied at that point we should calculate too time.

$$R_a = \frac{P}{2} = \frac{15}{2} = 7.5 \text{ KN}$$

$$R_b = \frac{P}{2} = \frac{15}{2} = 7.5 \text{ KN}$$

SFD

$$SF_A = -7.5 \text{ KN} + 15 \text{ KN} = 7.5 \text{ KN}$$

$$SF_B = -7.5 \text{ KN}$$

$$\begin{aligned} SF_C &= -7.5 \text{ KN} \\ &= -7.5 \text{ KN} + 15 \\ &= +7.5 \text{ KN} \end{aligned}$$

BMD

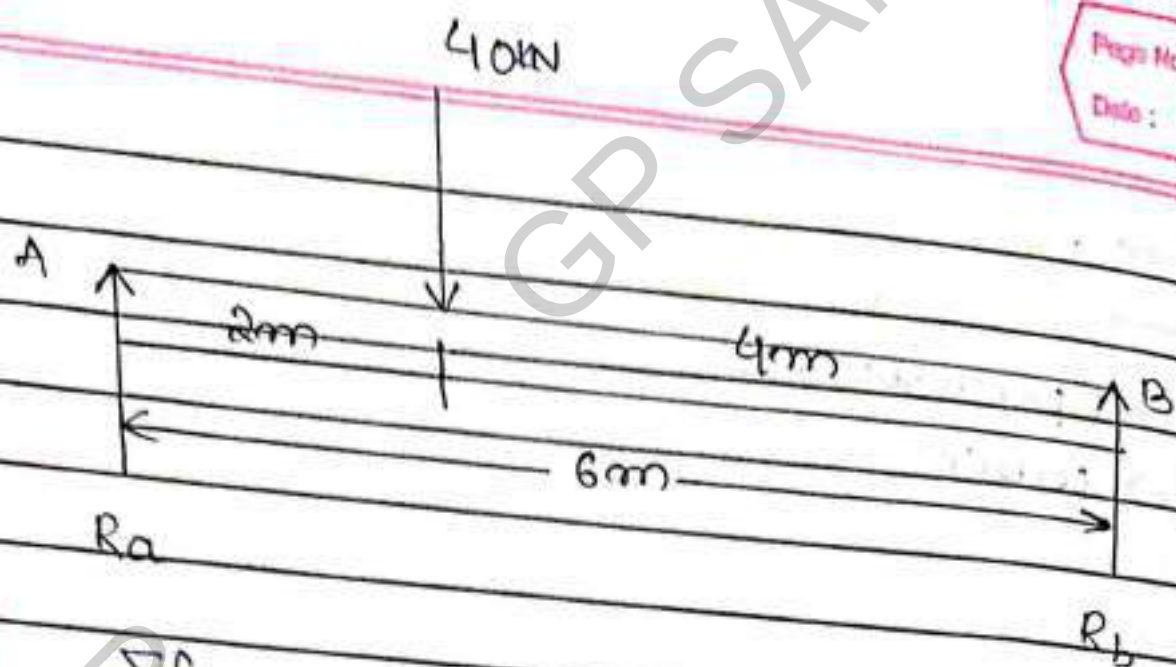
$$BM_A = 7.5 \times 8 - 15 \times 4 = 0 \text{ KN-m}$$

$$BM_B = 0 \text{ KN-m}$$

$$\begin{aligned} BM_C &= 7.5 \times 4 \\ &= +30 \text{ KN-m} \end{aligned}$$

$$\text{Max} = \frac{PL}{4}$$

$x^0 \rightarrow$  $x^1 \rightarrow$  $x^2 \rightarrow$ parabolic $x^3 \rightarrow$ cubic



$$\sum f_y = 0$$

$$\Rightarrow R_a + R_b - 40 = 0$$

$$\Rightarrow R_a + R_b = 40 \text{ kN}$$

$$M_A = 0$$

$$R_b \times 6 - 40 \times 2 = 0$$

$$R_b \times 6 - 80 = 0$$

$$R_b \times 6 = 80$$

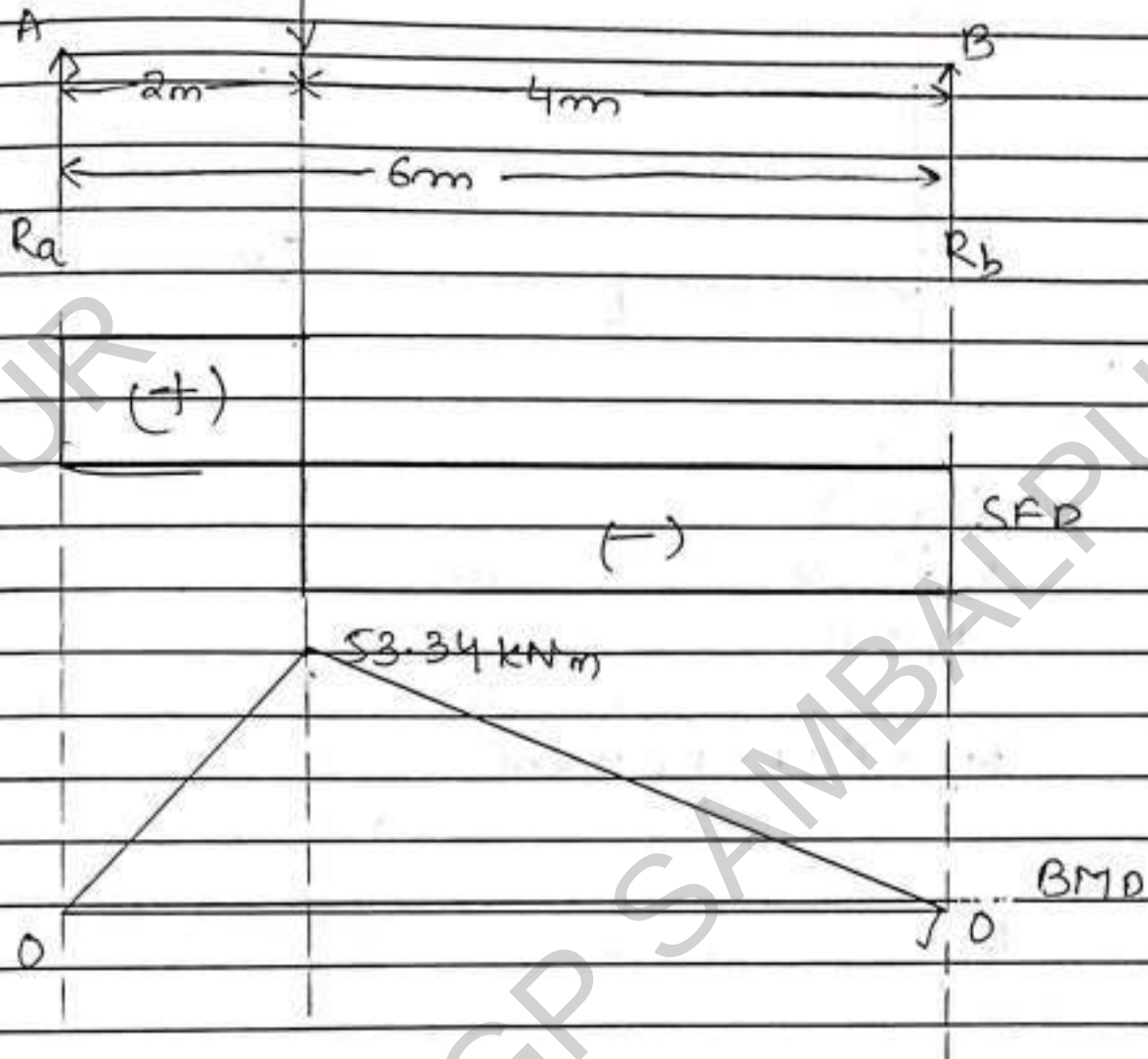
$$R_b = \frac{80}{6}$$

$$R_b = 13.33 \text{ kN}$$

$$R_a + 13.33 = 40$$

$$R_a = 40 - 13.33$$

$$R_a = 26.67 \text{ kN}$$

SFDBMD

$$SF_A = -13.33 + 40$$

$$= 26.67 \text{ kN}$$

$$SF_B = -13.33 \text{ kN}$$

$$SF_c = -13.33 + 40 = 26.7 \text{ kN}$$

(with load)

$$= -13.33 \text{ kN (without load)}$$

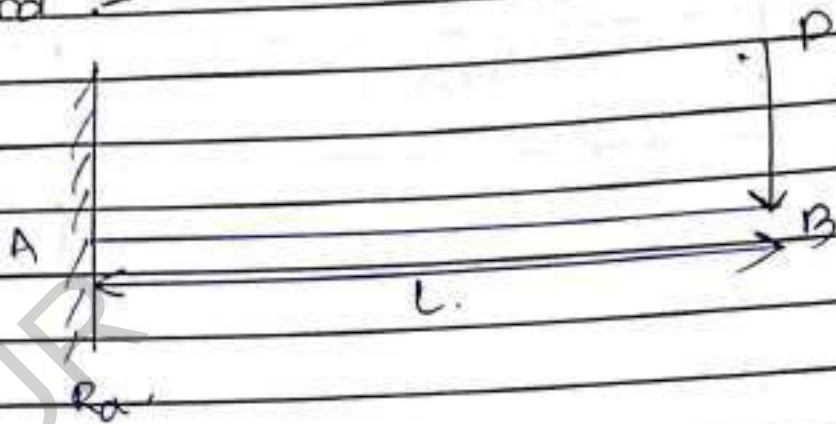
$$M_A = 0 \text{ kN-m}$$

$$M_B = 0 \text{ kN-m}$$

$$M_c = 26.67 \times 2$$

$$= 53.34 \text{ kNm}$$

Cantilever Beam with a point load at free end :-

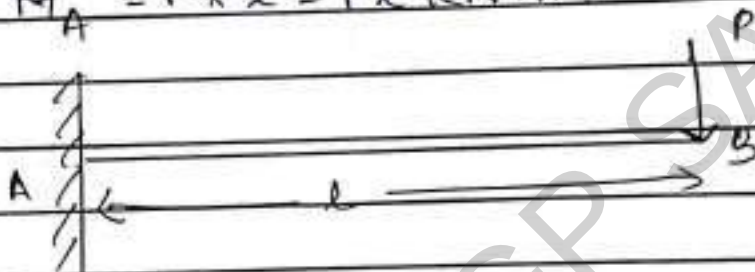


$$\sum F_y = 0$$

$$R_a - P = 0$$

$$R_a = P$$

$$M_A = P \times L = PL \text{ KNm}$$



(+)

SFD

(-)

BMD

SFD

$$SF_A = +P$$

$$SF_B = +P$$

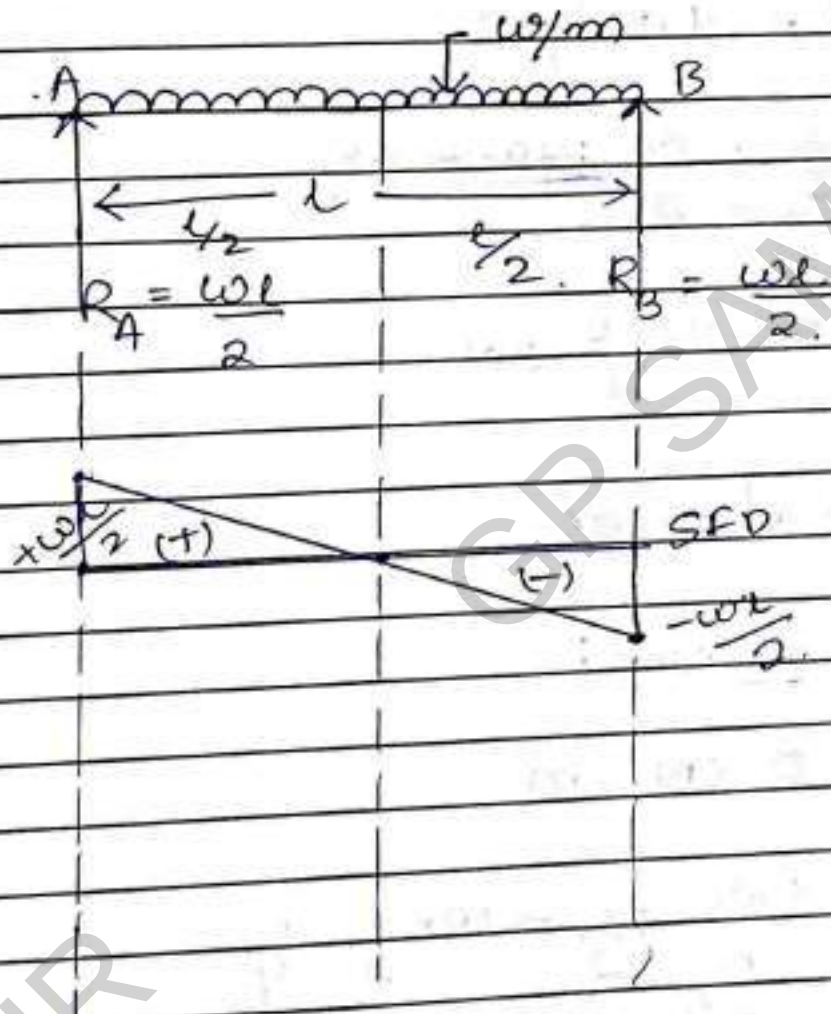
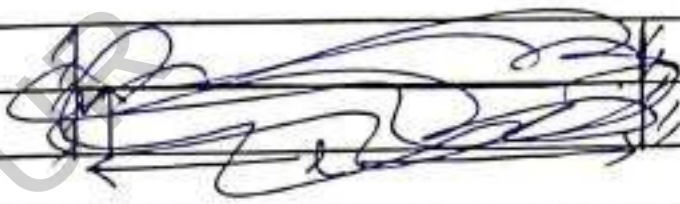
BMD

$$M_A = -Pl \text{ KNm}$$

$$M_B = 0.$$

Simply Supported Beam with U.D.L throughout the length.

UDL = uniformly distributed load.



Point = load $\times$ dis

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Shear force.

$$SF_B = -\frac{wl}{2} \text{ KN.}$$

$$SF_C = -\frac{wl}{2} + \frac{wl}{2}$$

$$= 0 \text{ KN}$$

$$SF_A = -\frac{wl}{2} + \frac{wl}{2}$$

$$= 0$$

$$= 0 + \frac{wl}{2} \text{ KN}$$

$l \rightarrow l'$ power.

Bending moment:-

$$BM_B = 0 \text{ KN-m.}$$

$$BM_C = \frac{+wl}{2} \times \frac{l}{2} - \frac{wl}{2} \times \frac{l}{4}$$

$$= \frac{wl^2}{4} - \frac{wl^2}{8}$$

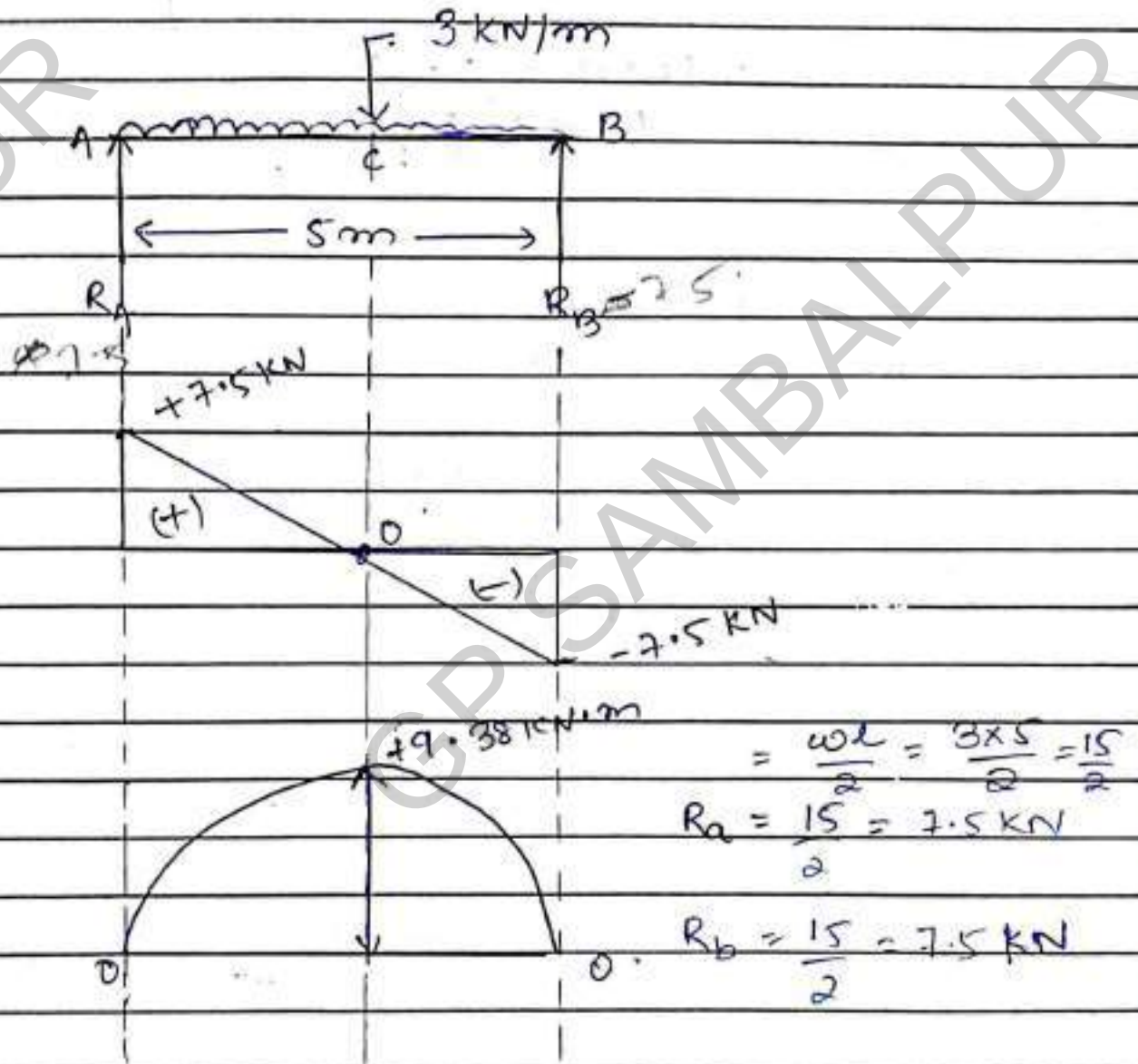
$$= \frac{2wl^2 - wl^2}{8}$$

$$= \frac{wl^2}{8}$$

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$$BM_A = 0 \text{ kN}$$

Draw the Shear force and BMD of a simply support beam having 5m span and subjected UDL of 3 kN/m.



Shear force

$$SF_B = -3 \times 2.5 = -7.5 \text{ kN}$$

$$SF_C = -3 \times 2.5 + 3 \times 2.5 = 0$$

$$SF_A = -3 \times 2.5 + 3 \times 5 = +7.5 \text{ kN}$$

$$\underline{BM}_B = 0$$

$$BM_A = 0$$

$$BM_C = 7.5 \times 2.5 - 7.5 \times \frac{5}{4}$$

$$= 18.75 - 9.375$$

$$= 9.38 \text{ KN/m}$$

Reactions

$$R_a = \frac{wl}{2} = \frac{3 \times 5}{2} = \frac{15}{2} = 7.5 \text{ KN}.$$

$$R_b = \frac{wl}{2} = \frac{3 \times 5}{2} = \frac{15}{2} = 7.5 \text{ KN}.$$

SFB

$$SF_B = -\frac{wl}{2} = -\frac{3 \times 5}{2} = -\frac{15}{2} = -7.5 \text{ KN}.$$

$$SF_A = -7.5 + 15 = +7.5.$$

$$SF_C = -7.5 + 7.5 = 0 \text{ KN}.$$

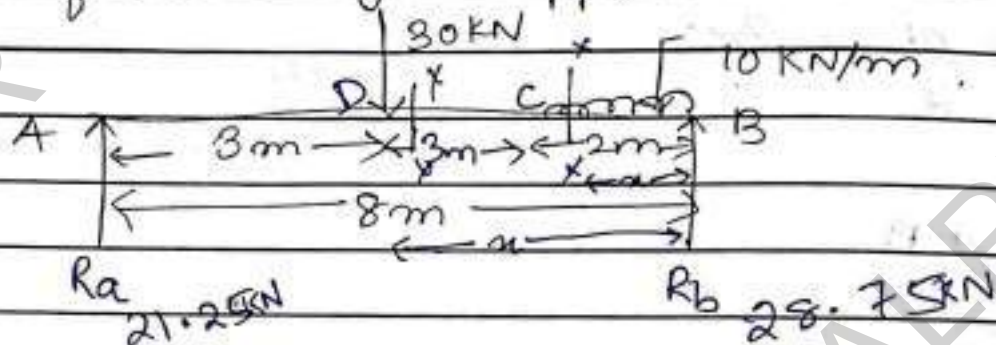
BMD

$$BM_B = R_b \times 0 = 0 \text{ KN-m}$$

$$\begin{aligned} BM_C &= +7.5 \times 2.5 - 3 \times 2.5 \times 1.25 \\ &= 9.375 \text{ KN-m} \end{aligned}$$

$$\begin{aligned} BM_A &= 7.5 \times 5 - 3 \times 5 \times 2.5 \\ &= 0 \text{ KN-m.} \end{aligned}$$

Draw the SFD and BMD of a simply supported beam having 8m span length subjected to 30kN point load at a distance 3m from left hand support and UDL of 10kN/m over 2m from the right support.



$$\sum F_y = 0$$

$$R_a + R_b - 30 - 20 = 0$$

$$R_a + R_b - 50 = 0$$

$$R_a + R_b = 50$$

$$\sum M_A = 0$$

$$R_b \times 8 - 20 \times 7 - 30 \times 3 = 0$$

$$R_b \times 8 - 140 - 90 = 0$$

$$8R_b - 230 = 0$$

$$R_b = \frac{230}{8}$$

$$= 28.75 \text{ kN}$$

$$R_a + R_b = 50.$$

$$R_a = 50 - 28.75$$

$$= 21.25 \text{ kN}$$

Shear force Equation:-

Portion BC (x from B)

$$S_x = -28.75 + 10x$$

$$S_B = -28.75 \text{ kN}$$

$$S_C = -28.75 + 20$$

$$= -8.75 \text{ kN}$$

Portion CD (x from D)

$$S_x = -28.75 + 20$$

$$= -8.75 \text{ kN}$$

$$S_C = -8.75 \text{ kN}$$

$$S_D = -8.75 \text{ kN}$$

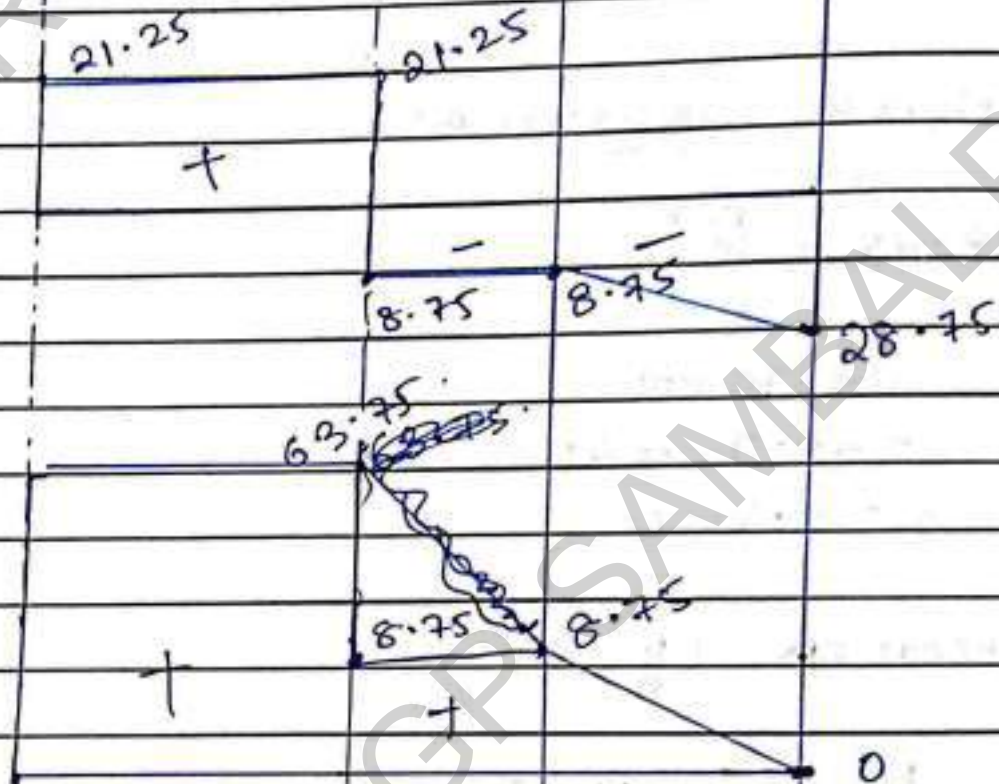
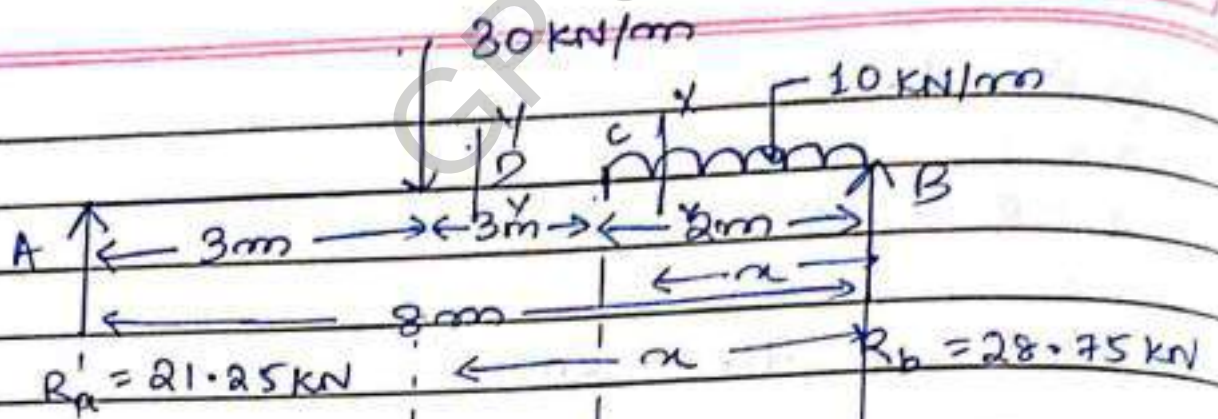
Portion DA (x from D)

$$S_x = -28.75 + 10 \times 2 + 30$$

$$= 21.25 \text{ kN}$$

$$S_D = 21.25 \text{ kN}$$

$$S_A = 21.25 \text{ kN}$$



Bending moment

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Portion BC (x from B)

$$M_B = 0 \text{ kNm}$$

$$M_E = 28.75 \times 1 - 20 \times 0.5 = 23.75$$

$$M_C = 28.75 - 20 \times 1 \times 2$$

$$M_C = 37.5 - 40 = -2.5 \text{ kNm}$$

Portion CD (x from B)

~~$$M_C = 28.75 \text{ kNm}$$~~

$$M_D = 28.75 \times 5 - 20 \times (4)$$

$$M_D = 63.75 \text{ kNm}$$

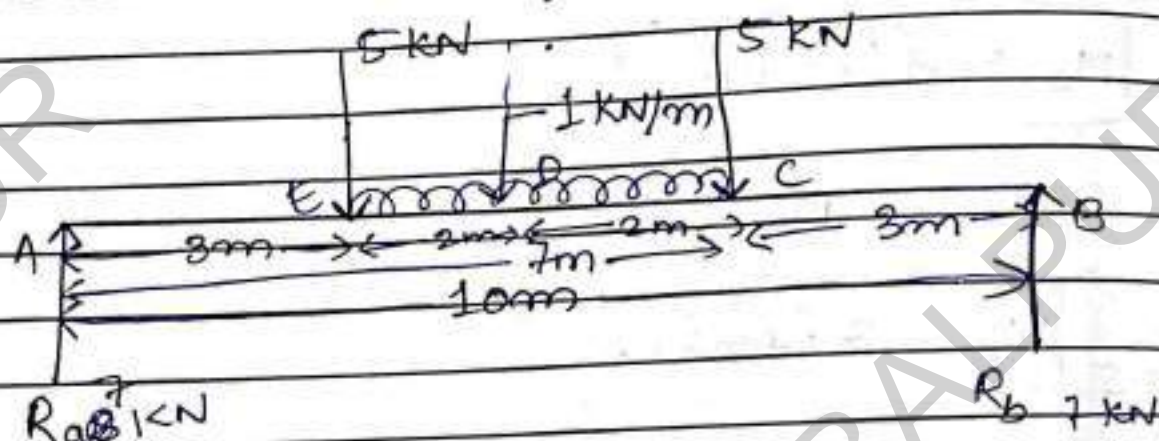
Portion DA

$$M_A = 28.75 \times 5 - 20 \times 4 = 63.75 \text{ kNm}$$

~~$$M_A = 28.75 \times 5 - 20 \times 4 - 20 \times 3 = 26.25 \text{ kNm}$$~~

~~$$M_A = 26.25 \text{ kNm}$$~~

A beam AB 10m long has supports at its ends A and B. It carries a point load of 5kN at 3m from A and a point load of 5kN at 7m from A and uniformly distributed load of 1kN/m between the point loads. Draw SFD and BMN for the beam.



$$\sum F_y = 0$$

$$R_a - 5 - 5 + R_b = 0$$

$$R_a + R_b - 10 = 0$$

$$R_a + R_b = 10 \quad (1)$$

$$\sum M_A = 0$$

$$R_b \times 10 - 5 \times 7 - 5 \times 3 = 0$$

$$10R_b - 35$$

$$R_b \times 10 - 5 \times 7 - 4 \times 5 - 5 \times 3 = 0$$

$$10R_b - 35 - 20 - 15 = 0$$

$$R_b = \frac{70}{10}$$

$$R_b = 7 \text{ kN}$$

A beam AB 10m long has supports at its ends A and B. It carries a point load 5 kN at 3m from A and a point load of 5 kN at 7m from A and a uniformly distributed load of 1 kN/m betn the point load. Draw SFD and BMD of the beam.



$$\sum f_y = 0$$

$$R_a + R_b - 5 - 5 - (1 \times 4) = 0.$$

$$R_a + R_b - 5 - 5 - 4 = 0$$

$$R_a + R_b = 14 \quad \text{--- (1)}$$

$$\sum M_A = 0$$

$$R_b \times 10 - 5 \times 7 - (1 \times 4) \times 5 - 5 \times 3 = 0$$

$$10R_b - 35 - 20 - 15 = 0.$$

$$10R_b = 70$$

$$\boxed{R_b = 7 \text{ kN}}$$

$$R_a + R_b = 14$$

$$R_a + 7 = 14$$

$$\boxed{R_a = 7 \text{ kN}}$$

Calculation for SFD

$$SF_B = -7 \text{ kN}$$

$$SF_C = (-7 + 5) \text{ kN} = -2 \text{ kN}$$

~~$$SF_D = -2 + (1 \times 4) = -2 \text{ kN}$$~~

~~$$SF_E = -2 + 5 = 3 \text{ kN}$$~~

$$SF_E = -2 + (1 \times 4) = 2 \text{ kN} \quad \checkmark$$

$$SF_F = 2 + 5 = 7 \text{ kN} \quad \checkmark$$

$$SF_A = 0 \text{ kN} \quad \checkmark$$

Bending Moment :-

$$BM_B = 0 \text{ kNm}$$

$$BM_C = 7 \times 3 = 21 \text{ kNm}$$

$$BM_E = 7 \times 7 - 5 \times 4 - 4 \times 2 = 21 \text{ kNm}$$

$$BM_A = 7 \times 10 - 5 \times 7 - 4 \times 5 - 5 \times 3 = 0 \text{ kNm}$$